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by parmenides51

1 Determine all positive integers k, ℓ, m and n, such that

$$\frac{1}{k!}+\frac{1}{\ell!}+\frac{1}{m!}=\frac{1}{n!}$$

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2 Given a triangle ABC, let P be a point inside the triangle such that $|BP| > |AP|, |BP| > |CP|$. Show that $\angle ABC < 90^o$

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3 Find all positive real numbers x, y, z, such that

$$x-\frac{1}{y^2}=y-\frac{1}{z^2}=z-\frac{1}{x^2}$$

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4 Towns A, B and C are connected with a telecommunications cable. If you for example want to send a message from A to B is assigned to either a direct line between A and B, or if necessary, a line via C. There are 43 lines between A and B, including those who go through C, and 29 lines between B and C, including those who go via A. How many lines, are there between A and C (including those who go via B)?

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5 Arne and Bertil play a game on an 11×11 grid. Arne starts. He has a game piece that is placed on the center od the grid at the beginning of the game. At each move he moves the piece one step horizontally or vertically. Bertil places a wall along each move any of an optional four squares. Arne is not allowed to move his piece through a wall. Arne wins if he manages to move the pice out of the board, while Bertil wins if he manages to prevent Arne from doing that. Who wins if from the beginning there are no walls on the game board and both players play optimally?

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6 How many functions $f:\mathbb{N}\to\mathbb{N}$ are there such that $f(0)=2011$, $f(1)=111$, and

$$f\left(\max\{x+y+2,xy\}\right)=\min\{f(x+y),f(xy+2)\}$$

for all non-negative integers x, y?
