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by parmenides51

- 1 The function f satisfies the condition

$$f(x+1) = \frac{1+f(x)}{1-f(x)}$$

for all real x , for which the function is defined. Determine $f(2012)$, if we know that $f(1000) = 2012$.

- 2 The number 201212200619 has a factor m such that $6 \cdot 10^9 < m < 6.5 \cdot 10^9$. Find m .
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- 3 The catheti AC and BC in a right-angled triangle ABC have lengths b and a , respectively. A circle centered at C is tangent to hypotenuse AB at point D . The tangents to the circle through points A and B intersect the circle at points E and F , respectively (where E and F are both different from D). Express the length of the segment EF in terms of a and b .
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- 4 Given that a is a real solution to the polynomial equation

$$nx^n - x^{n-1} - x^{n-2} - \dots - x - 1 = 0$$

where n is a positive integer, show that $a = 1$ or $-1 < a < 0$.

- 5 The vertices of a regular 13-gon are colored in three different colors. Show that there are three vertices which have the same color and are also the vertices of an isosceles triangle.
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- 6 A circle is inscribed in a trapezoid. Show that the diagonals of the trapezoid intersect at a point on the diameter of the circle perpendicular to the two parallel sides.
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