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by parmenides51

1 Given the acute triangle ABC. A diameter of the circumscribed circle of the triangle intersects the sides AC and BC, dividing the side BC in half. Show that the same diameter divides the side AC in a ratio of $1 : 3$, calculated from A, if and only if $\tan B = 2\tan C$.

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2 Determine all integer solutions to the equation $x^3 + y^3 + 2015 = 0$.

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3 Let a, b, c be positive real numbers. Determine the minimum value of the following expression

$$\frac{a^2 + 2b^2 + 4c^2}{b(a + 2c)}$$

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4 Solve the system of equations

$$\begin{cases} x\log x + y\log y + z\log x = 0 \\ \dfrac{\log x}{x} + \dfrac{\log y}{y} + \dfrac{\log z}{z} = 0 \end{cases}$$

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5 Given a finite number of points in the plane as well as many different rays starting at the origin. It is always possible to pair the points with the rays so that they parallell displaced rays starting in respective points do not intersect?

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6 Axel and Berta play the following games: On a board are a number of positive integers. One move consists of a player exchanging a number x on the board for two positive integers y and z (not necessarily different), such that $y + z = x$. The game ends when the numbers on the board are relatively coprime in pairs. The player who made the last move has then lost the game. At the beginning of the game, only the number 2015 is on the board. The two players make do their moves in turn and Berta begins. One of the players has a winning strategy. Who, and why?
