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by parmenides51

1 In a garden there is an L-shaped fence, see figure. You also have at your disposal two finished straight fence sections that are 13 m and 14 m long respectively. From point A you want to delimit a part of the garden with an area of at least 200 m^2 . Is it possible to do this?
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2 Determine whether the inequality

$$\left|\sqrt{x^2+2x+5}-\sqrt{x^2-4x+8}\right|<3$$

is valid for all real numbers x.

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3 The quadrilateral $ABCD$ is an isosceles trapezoid, where $AB \parallel CD$. The trapezoid is inscribed in a circle with radius R and center on side AB. Point E lies on the circumscribed circle and is such that $\angle DAE = 90^o$. Given that $\frac{AE}{AB} = \frac{3}{4}$, calculate the length of the sides of the isosceles trapezoid.

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4 Find all prime numbers p, for which the number $p + 1$ is equal to the product of all the prime numbers which are smaller than p.

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5 Peter wants to create a new multiplication table for the four numbers $1, 2, 3, 4$ in such a way that the product of two of them is also one of them. He wants also that $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ holds and that $ab \ne ac$ and $ba \ne ca$ and $b \ne c$. Peter is successful in constructing the new table. In his new table, $1 \cdot 3 = 2$ and $2 \cdot 2 = 4$. What is the product $3 \cdot 1$ according to Peter's table?

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6 Each cell in a 13×13 grid table is painted in black or white. Each move consists of choosing a subsquare of size either 2×2 or 9×9, and painting all white cells of the choosen subsquare black, and painting all its black cells white. It is always possible to get all cells of the original square black, after a finite number of such moves ?
