www.artofproblemsolving.com/community/c1995581
by parmenides51

1 Xenia and Yagve take turns in playing the following game: A coin is placed on the first box in a row of nine cells. At each turn the player may choose to move the coin forward one step, move the coin forward four steps, or move coin back two steps. For a move to be allowed, the coin must land on one of them of nine cells. The winner is one who gets to move the coin to the last ninth cell. Who wins, given that Xenia makes the first move, and both players play optimally?

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2 Let p be a prime number. Find all pairs of coprime positive integers (m, n) such that

$$\frac{p+m}{p+n} = \frac{m}{n} + \frac{1}{p^2}.$$

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3 Given the segments AB and CD not necessarily on the same plane. Point X is the midpoint of the segment AB, and the point Y is the midpoint of CD. Given that point X is not on line CD, and that point Y is not on line AB, prove that $2|XY| \le |AD| + |BC|$. When is equality achieved?

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4 Let D be the foot of the altitude towards BC in the triangle ABC. Let E be the intersection of AB with the bisector of angle $\angle C$. Assume that the angle $\angle AEC = 45^o$. Determine the angle $\angle EDB$.

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5 Find a costant C, such that

$$\frac{S}{ab+bc+ca} \le C$$

where a, b, c are the side lengths of an arbitrary triangle, and S is the area of the triangle.
(The maximal number of points is given for the best possible constant, with proof.)

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6 Let a, b, c, x, y, z be real numbers such that $x + y + z = 0$, $a + b + c \ge 0$, $ab + bc + ca \ge 0$. Prove that

$$ax^2 + by^2 + cz^2 \ge 0$$
