www.artofproblemsolving.com/community/c1995582
by parmenides51

**1** Let the $ABCD$ be a quadrilateral without parallel sides, inscribed in a circle. Let $P$ and $Q$ be the intersection points between the lines containing the quadrilateral opposite sides. Show that the bisectors to the angles at $P$ and $Q$ are parallel to the bisectors of the angles at the intersection point of the diagonals of the quadrilateral.

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**2** Find all functions $f : R \to R$ that satisfy $f(x) + 2f(\sqrt[3]{1 - x^3}) = x^3$ for all real $x$.

(Here $\sqrt[3]{x}$ is defined all over $R$.)

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**3** Let m be a positive integer. An $m$-*pattern* is a sequence of $m$ symbols of strict inequalities. An $m$-pattern is said to be *realized* by a sequence of $m + 1$ real numbers when the numbers meet each of the inequalities in the given order. (For example, the $5$-pattern $<, <, >, <, >$ is realized by the sequence of numbers $1, 4, 7, -3, 1, 0$.)
Given $m$, which is the least integer $n$ for which there exists any number sequence $x_1, ..., x_n$ such that each $m$-pattern is realized by a subsequence $x_{i_1}, ..., x_{i_{m+1}}$ with $1 \le i_1 < ... < i_{m+1} \le n$?

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**4** Find the least positive integer $n$ with the property:
Among arbitrarily $n$ selected consecutive positive integers, all smaller than $2018$, there is at least one that is divisible by its sum of digits .

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**5** In a triangle $ABC$, two lines are drawn that together trisect the angle at $A$. These intersect the side $BC$ at points $P$ and $Q$ so that $P$ is closer to $B$ and $Q$ is closer to $C$. Determine the smallest constant $k$ such that $|PQ| \le k(|BP| + |QC|)$, for all such triangles. Determine if there are triangles for which equality applies.

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**6** For which positive integers $n$ can the polynomial $p(x) = 1 + x^n + x^{2n}$ is written as a product of two polynomials with integer coefficients (of degree $\ge 1$)?

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