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by parmenides51

1 The siblings Robb, Arya and Sansa have received seven sealed bags from an unknown donor with varying number of beads. Six of the bags have labels indicating the number beads: $7, 9, 11, 13, 15, 18$, but the seventh bag lacks etiquette. The sensor has set certain requirements: Robb must have three bags and his sisters two bags each. In addition, Arya will have the bag that contains 7 beads. The bags should be distributed so that each of the siblings get the same number of pearls (this is possible according to the donor). How many pearls are there in the bag without a label, how many pearls are there in total and how should the bags be distributed?

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2 Segment AB is the diameter of a circle. Points C and D lie on the circle. The rays AC and AD intersect the tangent to the circle at point B at points P and Q, respectively. Show that points C, D, P and Q lie on a circle.

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3 There are two bowls on a table, one white and one black. In the white bowl there 2019 balls. Players A and B play a game where they make every other move (A begins).
One move consists is • to move one or your balls from one bowl to the other, or • to remove a ball from the white bowl,
with the condition that the resulting position (that is, the number of bullets in the two bowls) have not occurred before. The player who has no valid move to make loses.
Can any of the players be sure to win? If so, which one?

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4 Let Ω be a circle disk with radius 1. Determine the minimum r that has the following property: You can select three points on Ω so that each circle disk located in Ω and has a radius greater than r contains at least one of the three points.

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5 Let f be a function that is defined for all positive integers and whose values are positive integers. For f it also holds that $f(n+1) > f(n)$ and $f(f(n)) = 3n$, for each positive integer n. Calculate $f(2019)$.

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6 Is there an infinite sequence of positive integers $\{a_n\}_{n=1}^{\infty}$ which contains each positive integer exactly once and is such that the number $a_n + a_{n+1}$ is a perfect square for each n?
