www.artofproblemsolving.com/community/c1995584
by parmenides51

1 How many of the numbers $1 \cdot 2 \cdot 3$, $2 \cdot 3 \cdot 4$,..., $2020 \cdot 2021 \cdot 2022$ are divisible by 2020?

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2 The medians of the sides AC and BC in the triangle ABC are perpendicular to each other. Prove that $\frac{1}{2} < \frac{|AC|}{|BC|} < 2$.

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3 Determine all bounded functions $f : R \to R$, such that $f(f(x) + y) = f(x) + f(y)$, for all real x, y.

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4 Which is the least positive integer n for which it is possible to find a (non-degenerate) n-gon with sidelengths $1, 2, ..., n$, and where all vertices have integer coordinates?

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5 Find all integers a such that there is a prime number of $p \ge 5$ that divides $\binom{p-1}{2} + \binom{p-1}{3} a + \binom{p-1}{4} a^2$+ ...+$\binom{p-1}{p-3} a^{p-5}$.

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6 A finite set of *axis parallel* cubes in space has the property of each point of the room is located in a maximum of M different cubes. Show that you can divide the amount of cubes in $8(M-1)+1$ subsets (or less) with the property that the cubes in each subset lacks common points. (An axis parallel cube is a cube whose edges are parallel to the coordinate axes.)
