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– **Grade 8**

– **Day 1**

Problem 1 Solve the system of equations $2|x|+|y|=1$, $\lfloor |x| \rfloor + \lfloor 2|y| \rfloor = 2$.

Problem 2 Is it possible to write numbers in the cells of a 7×7 board in such a way that the sum of numbers in every 2×2 or 3×3 square is divisible by 1999, but the sum of all numbers in the board is not divisible by 1999?

Problem 3 Is there a 2000-digit number which is a perfect square and 1999 of whose digits are fives?

– **Day 2**

Problem 4 Same as part (b) of Grade 11 Problem 5

Problem 5 Let N be the point inside a rhombus $ABCD$ such that the triangle BNC is equilateral. The bisector of $\angle ABN$ meets the diagonal AC at K. Show that $BK=KN+ND$.

Problem 6 Consider the figure consisting of 19 hexagonal cells, as shown in the picture. At the cell A, there is a piece that is allowed to move one cell up, up-right, or down-right. How many ways are there for the piece to reach the cell B, not passing through the cell C?
`https://services.artofproblemsolving.com/download.php?id=YXR0YWNobWVudHMvZi9mL2Y4MWY5NGMw`
`=\&rn=VWtyYWluZSAxOTk5LnBuZw==`

– **Grade 9**

– **Day 1**

Problem 1 Describe the region in the coordinate plane defined by $|x^2+xy|\ge |x^2-xy|$.

Problem 2 Let x and y be positive real numbers with $(x-1)(y-1)\ge 1$. Prove that for sides a,b,c of an arbitrary triangle we have $a^2x+b^2y>c^2$.

Problem 3 Show that the number $9999999+1999000$ is composite.

Problem 4 The bisectors of angles A,B,C of a triangle ABC intersect the circumcircle of the triangle at A_1,B_1,C_1, respectively. Let P be the intersection of the lines B_1C_1 and AB, and Q be the intersection of the lines B_1A_1 and BC. Show how to construct the triangle ABC by a ruler and a compass, given its circumcircle, points P and Q, and the halfplane determined by PQ in which point B lies.

– Day 2

Problem 5 Solve the equation $\lfloor x \rfloor + \frac{1999}{\lfloor x \rfloor} = \{x\} + \frac{1999}{\{x\}}$.

Problem 6 Find all pairs (k, l) of positive integers such that $\frac{k^l}{l^k} = \frac{k!}{l!}$.

Problem 7 Let M be a fixed point inside a given circle. Two perpendicular chords AC and BD are drawn through M, and K and L are the midpoints of AB and CD, respectively. Prove that the quantity $AB^2 + CD^2 - 2KL^2$ is independent of the chords AC and BD.

Problem 8 A sequence of natural numbers (a_n) satisfies $a_{a_n} + a_n = 2n$ for all $n \in \mathbb{N}$. Prove that $a_n = n$.

– Grade 10

– Day 1

Problem 1 Solve the equation $\sin x \sin 2x \sin 3x + \cos x \cos 2x \cos 3x = 1$.

Problem 2 Let M be a point inside a triangle ABC. The line through M parallel to AC meets AB at N and BC at K. The lines through M parallel to AB and BC meet AC at D and L, respectively. Another line through M intersects the sides AB and BC at P and R respectively such that $PM = MR$. Given that the area of $\triangle ABC$ is S and that $\frac{CK}{CB} = a$, compute the area of $\triangle PQR$.

Problem 3 Missing

Problem 4 Two players alternately write integers on a blackboard as follows: the first player writes a_1 arbitrarily, then the second player writes a_2 arbitrarily, and thereafter a player writes a number that is equal to the sum of the two preceding numbers. The player after whose move the obtained sequence contains terms such that $a_i - a_j$ and $a_{i+1} - a_{j+1}$ $(i \neq j)$ are divisible by 1999, wins the game. Which of the players has a winning strategy?

– Day 2

Problem 5 Evaluate

$$\lfloor \pi \rfloor + \left\lfloor \frac{\lfloor 2\pi \rfloor}{2} \right\rfloor + \left\lfloor \frac{\lfloor 3\pi \rfloor}{3} \right\rfloor + \ldots + \left\lfloor \frac{\lfloor 1999\pi \rfloor}{1999} \right\rfloor .$$

Problem 6 Solve the equation $m^3 - n^3 = 7mn + 5$ in positive integers.

Problem 7 If x_1, x_2, ..., $x_6 \in [0, 1]$, prove that the cyclic sum of

$\frac{x_1^3}{x_2^5+x_3^5+x_4^5+x_5^5+x_6^5+5}$ is less than $\frac{3}{5}$.

Problem 8 Let AA_1, BB_1, CC_1 be the altitudes of an acute-angled triangle ABC, and let O be an arbitrary interior point. Let M, N, P, Q, R, S be the feet of the perpendiculars from O to the lines $AA_1, BC, BB_1, CA, CC_1, AB$, respectively. Prove that the lines MN, PQ, RS are concurrent.

– Grade 11

– Day 1

Problem 1 Solve the equation

$$(\sin x)^{1998} + (\cos x)^{-1999} = (\cos x)^{1998} + (\sin x)^{-1999}.$$

Problem 2 Find all values of the parameter k for which the system of inequalities

$$ky^2 + 4ky - 2x + 6k + 3 \leq 0$$
$$kx^2 - 2y - 2kx + 3k - 3 \leq 0$$

has a unique solution.

Problem 3 All faces of a parallelepiped $ABCDA_1B_1C_1D_1$ are rhombi, and their angles at A are all equal to α. Points M, N, P, Q are selected on the edges A_1B_1, DC, BC, A_1D_1, respectively, such that $A_1M = BP$ and $DN = A_1Q$. Find the angle between the intersection lines of the plane A_1BD with the planes AMN and APQ.

Problem 4 Same as Grade 10 Problem 4.

– Day 2

Problem 5 Can the number (a)19991998 (b)19991999 be written in the form $n^4 + m^3 - m$, where n, m are integers?

Problem 6 Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(xy) + f(xz) - f(x)f(yz) \geq 1 \qquad \text{for all } x, y, z.$$

Problem 7 Suppose that the function $f(x) = \tan(a_1x + 1) + \ldots + \tan(a_{10}x + 1)$ has the period $T > 0$, where $a_1, \ldots, a_{10}$ are positive numbers. Prove that

$$T \geq \frac{\pi}{10} \min\left\{\frac{1}{a_1}, \ldots, \frac{1}{a_{10}}\right\}.$$

Problem 8 Same as Grade 10 Problem 8.
