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Problem 1 Consider a regular n -gon $A_1A_2 \dots A_n$ with area S . Let us draw the lines l_1, l_2, \dots, l_n perpendicular to the plane of the n -gon at A_1, A_2, \dots, A_n respectively. Points B_1, B_2, \dots, B_n are selected on lines l_1, l_2, \dots, l_n respectively so that:

- (i) B_1, B_2, \dots, B_n are all on the same side of the plane of the n -gon;
- (ii) Points B_1, B_2, \dots, B_n lie on a single plane;
- (iii) $A_1B_1 = h_1, A_2B_2 = h_2, \dots, A_nB_n = h_n$.

Express the volume of polyhedron $A_1A_2 \dots A_nB_1B_2 \dots B_n$ as a function in S, h_1, \dots, h_n .

Problem 1 Given a natural number k , find the smallest natural number C such that

$$\frac{C}{n+k+1} \binom{2n}{n+k}$$

is an integer for every integer $n \geq k$.

Problem 3 Numbers $1, 2, \dots, 1997^2$ are written in the cells of a 1997×1997 table. It is allowed to apply the following transformations: exchange places of any two rows or any two columns, or reverse a row or column. (When a row or column is reversed, the first and last entry exchange their positions, so do the second and second last, etc.) Is it possible that, after finitely many such transformations, arbitrary two numbers exchange their positions and no other number changes its position?
