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by jasperE3

**Problem 1** Let  $\mathfrak{F} = \{A_1, A_2, \dots, A_n\}$  be a collection of subsets of the set  $S = \{1, 2, \dots, n\}$  satisfying the following conditions:

- (a) Any two distinct sets from  $\mathfrak{F}$  have exactly one element in common;
- (b) each element of  $S$  is contained in exactly  $k$  of the sets in  $\mathfrak{F}$ .

Can  $n$  be equal to 1996?

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**Problem 2** Let there be given a set of 1996 equal circles in the plane, no two of them having common interior points. Prove that there exists a circle touching at most three other circles.

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**Problem 3** The sequence  $\{x_n\}$  is given by

$$x_n = \frac{1}{4} \left( (2 + \sqrt{3})^{2n-1} + (2 - \sqrt{3})^{2n-1} \right), \quad n \in \mathbb{N}.$$

Prove that each  $x_n$  is equal to the sum of squares of two consecutive integers.

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