2021 Benelux Mathematical Olympiad. Please note: problems are not ordered by difficulty.

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by Jetze

1 (a) Prove that for all $a, b, c, d \in \mathbb{R}$ with $a+b+c+d=0$,

$$\max(a,b)+\max(a,c)+\max(a,d)+\max(b,c)+\max(b,d)+\max(c,d) \geqslant 0.$$

(b) Find the largest non-negative integer k such that it is possible to replace k of the six maxima in this inequality by minima in such a way that the inequality still holds for all $a, b, c, d \in \mathbb{R}$ with $a+b+c+d=0$.

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2 Pebbles are placed on the squares of a 2021×2021 board in such a way that each square contains at most one pebble. The pebble set of a square of the board is the collection of all pebbles which are in the same row or column as this square. (A pebble belongs to the pebble set of the square in which it is placed.) What is the least possible number of pebbles on the board if no two squares have the same pebble set?

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3 A cyclic quadrilateral $ABXC$ has circumcentre O. Let D be a point on line BX such that $AD = BD$. Let E be a point on line CX such that $AE = CE$. Prove that the circumcentre of triangle $\triangle DEX$ lies on the perpendicular bisector of OA.

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4 A sequence $a_1, a_2, a_3, \ldots$ of positive integers satisfies $a_1 > 5$ and $a_{n+1} = 5 + 6 + \cdots + a_n$ for all positive integers n. Determine all prime numbers p such that, regardless of the value of a_1, this sequence must contain a multiple of p.
