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by jasperE3

Problem 1 Circles k and l intersect at points P and Q. Let A be an arbitrary point on k distinct from P and Q. Lines AP and AQ meet l again at B and C. Prove that the altitude from A in triangle ABC passes through a point that does not depend on A.

Problem 2 Let a,b,c,m be integers, where $m>1$. Prove that if

$$a^n+bn+c\equiv 0\pmod m$$

for each natural number n, then $b^2\equiv 0\pmod m$. Must $b\equiv 0\pmod m$ also hold?

Problem 3 A sequence (x_n) satisfies $x_{n+1}=\frac{x_n^2+a}{x_{n-1}}$ for all $n\in\mathbb N$. Prove that if x_0,x_1, and $\frac{x_0^2+x_1^2+a}{x_0x_1}$ are integers, then all the terms of sequence (x_n) are integers.
