

www.artofproblemsolving.com/community/c2014379

by jasperE3

Problem 1 Find all integers x, y, z such that $x^2(x^2 + y) = y^{z+1}$.

Problem 2 Let k_0 be a unit semi-circle with diameter AB . Assume that k_1 is a circle of radius $r_1 = \frac{1}{2}$ that is tangent to both k_0 and AB . The circle k_{n+1} of radius r_{n+1} touches k_n, k_0 , and AB . Prove that:

(a) For each $n \in \{2, 3, \dots\}$ it holds that $\frac{1}{r_{n+1}} + \frac{1}{r_{n-1}} = \frac{6}{r_n} - 4$.

(b) $\frac{1}{r_n}$ is either a square of an even integer, or twice a square of an odd integer.

Problem 3 Let F be the collection of subsets of a set with n elements such that no element of F is a subset of another of its elements. Prove that

$$|F| \leq \binom{n}{\lfloor n/2 \rfloor}.$$
