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by jasperE3

**Problem 1** Determine the set of all real numbers  $\alpha$  with the following property: For each positive  $c$  there exists a rational number  $\frac{m}{n}$  ( $m \in \mathbb{Z}, n \in \mathbb{N}$ ) different than  $\alpha$  such that

$$\left| \alpha - \frac{m}{n} \right| < \frac{c}{n}.$$

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**Problem 2** Determine all 6-tuples  $(p, q, r, x, y, z)$  where  $p, q, r$  are prime, and  $x, y, z$  natural numbers such that  $p^{2x} = q^y r^z + 1$ .

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**Problem 3** Assume that the equality  $2BC = AB + AC$  holds in  $\triangle ABC$ . Prove that:

- (a) The vertex  $A$ , the midpoints  $M$  and  $N$  of  $AB$  and  $AC$  respectively, the incenter  $I$ , and the circumcenter  $O$  belong to a circle  $k$ .
  - (b) The line  $GI$ , where  $G$  is the centroid of  $\triangle ABC$  is a tangent to  $k$ .
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