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by jasperE3

Problem 1 Assume that a is a given irrational number.

(a) Prove that for each positive real number ϵ there exists at least one integer $q \geq 0$ such that $aq - \lfloor aq \rfloor < \epsilon$.
(b) Prove that for given $\epsilon > 0$ there exist infinitely many rational numbers $\frac{p}{q}$ such that $q > 0$ and $\left|a - \frac{p}{q}\right| < \frac{\epsilon}{q}$.

Problem 2 Given two directly congruent triangles ABC and $A'B'C'$ in a plane, assume that the circles with centers C and C' and radii CA and $C'A'$ intersect. Denote by $\mathcal{M}$ the transformation that maps $\triangle ABC$ to $\triangle A'B'C'$. Prove that $\mathcal{M}$ can be expressed as a composition of at most three rotations in the following way: The first rotation has the center in one of A, B, C and maps $\triangle ABC$ to $\triangle A_1B_1C_1$; The second rotation has the center in one of A_1, B_1, C_1, and maps $\triangle A_1B_1C_1$ to $\triangle A_2B_2C_2$; The third rotation has the center in one of A_2, B_2, C_2 and maps $\triangle A_2B_2C_2$ to $\triangle A'B'C'$.

Problem 3 Let S be a set of n points $P_1, P_2, \dots, P_n$ in a plane such that no three of the points are collinear. Let α be the smallest of the angles $\angle P_iP_jP_k$ ($i \neq j \neq k \neq i, i, j, k \in \{1, 2, \dots, n\}$). Find $\max_S \alpha$ and determine those sets S for which this maximal value is attained.
