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Problem 1 Given real numbers a_i, b_i $(i = 1, 2, \dots, n)$ such that

$$\begin{aligned}
&a_1 \geq a_2 \geq \dots \geq a_n > 0,\\
&b_1 \geq a_1,\\
&b_1 b_2 \geq a_1 a_2,\\
&\vdots\\
&b_1 b_2 \cdots b_n \geq a_1 a_2 \cdots a_n,
\end{aligned}$$

prove that $b_1 + b_2 + \dots + b_n \geq a_1 + a_2 + \dots + a_n$.

Problem 2 Let $f(x)$ and $g(x)$ be degree n polynomials, and $x_0, x_1, \dots, x_n$ be real numbers such that

$$f(x_0) = g(x_0), f'(x_1) = g'(x_1), f''(x_2) = g''(x_2), \dots, f^{(n)}(x_n) = g^{(n)}(x_n).$$

Prove that $f(x) = g(x)$ for all x.

Problem 3 Points A and B move with a constant speed along lines a and b. Two corresponding positions of these points A_1, B_1, and A_2, B_2 are known. Find the position of A and B for which the length of AB is minimal.

Problem 4 Let a and b be two natural numbers such that $a < b$. Prove that in each set of b consecutive positive integers there are two numbers whose product is divisible by ab.

Problem 5 Prove that the product of the sines of two opposite dihedrals in a tetrahedron is proportional to the product of the lengths of the edges of these dihedrals.

Problem 6 Let E be the set of n^2+1 closed intervals on the real axis. Prove that there exists a subset of $n+1$ intervals that are monotonically increasing with respect to inclusion, or a subset of $n+1$ intervals none of which contains any other interval from the subset.
