by jasperE3

Problem 1 Given 6 points in a plane, assume that each two of them are connected by a segment. Let D be the length of the longest, and d the length of the shortest of these segments. Prove that $\frac{D}{d} \geq \sqrt{3}$.

Problem 2 Let $n > 3$ be a positive integer. Prove that n is prime if and only if there exists a positive integer α such that $n! = n(n-1)(\alpha n + 1)$.

Problem 3 Each side of a triangle ABC is divided into three equal parts, and the middle segment in each of the sides is painted green. In the exterior of $\triangle ABC$ three equilateral triangles are constructed, in such a way that the three green segments are sides of these triangles. Denote by A', B', C' the vertices of these new equilateral triangles that dont belong to the edges of $\triangle ABC$, respectively. Let A'', B'', C'' be the points symmetric to A', B', C' with respect to BC, CA, AB.

(a) Prove that $\triangle A'B'C'$ and $\triangle A''B''C''$ are equilateral.
(b) Prove that $ABC, A'B'C'$, and $A''B''C''$ have a common centroid.

Problem 4 If a polynomial of degree n has integer values when evaluated in each of $k, k+1, \dots, k+n$, where k is an integer, prove that the polynomial has integer values when evaluated at each integer x.

Problem 5 Let n be an integer greater than 1. Let $x \in \mathbb{R}$.

(a) Evaluate $S(x,n) = \sum (x+p)(x+q)$, where the summation is over all pairs (p,q) of different numbers from $\{1, 2, \dots, n\}$.
(b) Do there exist integers x, n for which $S(x,n) = 0$?

Problem 6 Prove that the incenter coincides with the circumcenter of a tetrahedron if and only if each pair of opposite edges are of equal length.
