International Zhautykov Olympiad 2016

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Day 1

1 A quadrilateral $ABCD$ is inscribed in a circle with center O. It's diagonals meet at M.The circumcircle of ABM intersects the sides AD and BC at N and K respectively. Prove that areas of $NOMD$ and $KOMC$ are equal.

2 $a_1, a_2, ..., a_{100}$ are permutation of $1, 2, ..., 100$. $S_1 = a_1, S_2 = a_1 + a_2, ..., S_{100} = a_1 + a_2 + ... + a_{100}$Find the maximum number of perfect squares from S_i

3 There are 60 towns in $Graphland$ every two countries of which are connected by only a directed way. Prove that we can color four towns to red and four towns to green such that every way between green and red towns are directed from red to green

Day 2

1 Find all $k > 0$ for which a strictly decreasing function $g : (0; +\infty) \to (0; +\infty)$ exists such that $g(x) \geq kg(x + g(x))$ for all positive x.

2 A convex hexagon $ABCDEF$ is given such that $AB||DE$, $BC||EF$, $CD||FA$. The point M, N, K are common points of the lines BD and AE, AC and DF, CE and BF respectively. Prove that perpendiculars drawn from M, N, K to lines AB, CD, EF respectively concurrent.

3 We call a positive integer q a $convenient \quad denominator$ for a real number α if $|\alpha - \frac{p}{q}| < \frac{1}{10q}$ for some integer p. Prove that if two irrational numbers α and β have the same set of convenient denominators then either $\alpha + \beta$ or $\alpha - \beta$ is an integer.