Kosovo National Mathematical Olympiad 2010

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–	Grade 9
1	Solve the equation $\|x+1\|-\|x-1\|=2$.
2	Someones age is equal to the sum of the digits of his year of birth. How old is he and when was he born, if it is known that he is older than 11. P.s. the current year in the problem is 2010.
3	Prove that in any polygon, there exist two sides whose radio is less than 2.(Essentialy if $a_1\geq a_2\geq ...\geq a_n$ are the sides of a polygon prove that there exist $i,j\in \{1,2,..,n\}$ so that $i<j$ and $\frac{a_i}{a_j}<2$).
4	Prove that $\sqrt{3}$ is irrational.
5	Let x,y be positive real numbers such that $x+y=1$. Prove that $\left(1+\frac{1}{x}\right)\left(1+\frac{1}{y}\right)\geq 9$.
–	Grade 10
1	Find the graph of the function $y=\|2^{\|x\|}-1\|$.
2	The equation is given $x^2-(m+3)x+m+2=0$. If x_1 and x_2 are its solutions find all m such that $\frac{x_1}{x_1+1}+\frac{x_2}{x_2+1}=\frac{13}{10}$.
4	Let a,b,c be non negative integers. Suppose that c is even and $a^5+4b^5=c^5$. Prove that $b=0$.
5	same as 9.5
–	Grade 11
1	Solve the inequation $\sqrt{3-x}-\sqrt{x+1}>\frac{1}{2}$.
2	Let $a_1,a_2,...,a_n$ be an arithmetic progression of positive real numbers. Prove that $\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+...+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}=\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$.
3	Arrange the numbers $\cos 2, \cos 4, \cos 6$ and $\cos 8$ from the biggest to the smallest where $2,4,6,8$ are given as radians.

4 Prove that $\sqrt[3]{5}$ is irrational.

5 same as 9.5

– Grade 12

1 If the real function $f(x)=\cos x+\sum_{i=1}^{n}\cos(a_ix)$
is periodic, prove that $a_i, i\in\{1,2,...,n\}$, are rational numbers.

2 The set $S\subseteq\mathbb{R}$ is given with the properties: $(a)\mathbb{Z}\subset S$, $(b)(\sqrt{2}+\sqrt{3})\in S$, (c) If $x,y\in S$ then $x+y\in S$, and (d) If $x,y\in S$ then $x\cdot y\in S$.
Prove that $(\sqrt{2}+\sqrt{3})^{-1}\in S$.

3 Let $n\in\mathbb{N}$. Prove that the polynom $p(x)=x^{2n}-2x^{2n-1}+3x^{2n-2}-...-2nx+2n+1$
doesn't have real roots.

4 Let $(p_1,p_2,...,p_n)$ be a random permutation of the set $\{1,2,...,n)$. If n is odd, prove that the product $(p_1-1)\cdot(p_2-2)\cdot...\cdot(p_n-n)$
is an even number.
@below fixed.

5 same as 9.5