

Kosovo National Mathematical Olympiad 2009

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– Grade 10

1 Find the graph of the function $y = x - |x + x^2|$

2 If x_1 and x_2 are the solutions of the equation $x^2 - (m + 3)x + m + 2 = 0$ Find all real values of m such that the following inequations are valid $\frac{1}{x_1} + \frac{1}{x_2} > \frac{1}{2}$ and $x_1^2 + x_2^2 < 5$

3 Prove that $\sqrt{2}$ is irrational.

4 Prove that if in the product of four consecutive natural numbers we add 1, we get a perfect square.

5 In a circle four distinct points are fixed and each of them is assigned with a real number. Let those numbers be x_1, x_2, x_3, x_4 such that $x_1 + x_2 + x_3 + x_4 > 0$. Now we define a game with these numbers: If one of them, i.e. x_i , is a negative number, the player makes a move by adding the number x_i to his neighbors and changes the sign of the chosen number. The game ends when all the numbers are negative. Prove that this game ends in a finite number of steps.

– Grade 11

1 Find the graph of the function $y = 1 - |1 - \sin x|$.

2 Solve the equation: $x^2 + 2x\cos(x - y) + 1 = 0$

3 Let $n \geq 2$ be an integer. n is a prime if it is only divisible by 1 and n . Prove that there are infinitely many prime numbers.

4 Prove that $n^{11} - n$ is divisible by 11.

– Grade 12

1 Find the graph of the function $y = x + |1 - x^3|$.

2 Let p be a prime number and n a natural one. How many natural numbers are between 1 and p^n that are relatively prime with p^n ?

- 3** Let a, b and c be the sides of a triangle, prove that $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} < 2$.
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- 4** (a) Let a_1, a_2, a_3 be three real numbers. Prove that $(a_1 - a_2)(a_1 - a_3) + (a_2 - a_1)(a_2 - a_3) + (a_3 - a_1)(a_2 - a_2) \geq 0$. (b) Prove that the inequality above doesn't hold if we use four number instead of three.
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