**2015, Bay Area Mathematical Olympiad 8/12**

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by adihaya

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| – | February 24th, 2015 |
| – | BAMO-8 |
| 1 | There are $7$ boxes arranged in a row and numbered $1$ through $7$. You have a stack of $2015$ cards, which you place one by one in the boxes. The first card is placed in box#$1$, the second in box#$2$, and so forth up to the seventh card which is placed in box#$7$. You then start working back in the other direction, placing the eighth card in box#$6$, the ninth in box#$5$, up to the thirteenth card being placed in box#$1$. The fourteenth card is then placed in box#$2$, and this continues until every card is distributed. What box will the last card be placed in? |
| 2 | Members of a parliament participate in various committees. Each committee consists of at least $2$ people, and it is known that every two committees have at least one member in common. Prove that it is possible to give each member a colored hat (hats are available in black, white or red) so that every committee contains at least $2$ members with different hat colors. |
| 3 | Which number is larger, $A$ or $B$, where $$A = \frac{1}{2015}(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{2015})$$ and $$B = \frac{1}{2016}(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{2016})\ ?$$ Prove your answer is correct. |
| 4 | In a quadrilateral, the two segments connecting the midpoints of its opposite sides are equal in length. Prove that the diagonals of the quadrilateral are perpendicular.<br>(In other words, let $M, N, P,$ and $Q$ be the midpoints of sides $AB, BC, CD,$ and $DA$ in quadrilateral $ABCD$. It is known that segments $MP$ and $NQ$ are equal in length. Prove that $AC$ and $BD$ are perpendicular.) |
| – | BAMO-12 |
| 1 | Same as BAMO-8#3. |
| 2 | Same as BAMO-8#4. |

**3** Let $k$ be a positive integer. Prove that there exist integers $x$ and $y$, neither of which is divisible by $3$, such that $x^2 + 2y^2 = 3^k$.

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**4** Let $A$ be a corner of a cube. Let $B$ and $C$ the midpoints of two edges in the positions shown on the figure below:

`http://i.imgur.com/tEODnV0.png`

The intersection of the cube and the plane containing $A, B,$ and $C$ is some polygon, $P$.

- How many sides does $P$ have? Justify your answer.
- Find the ratio of the area of $P$ to the area of $\triangle ABC$ and prove that your answer is correct.

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**5** We are given $n$ identical cubes, each of size $1\times1\times1$. We arrange all of these $n$ cubes to produce one or more congruent rectangular solids, and let $B(n)$ be the number of ways to do this.

For example, if $n = 12$, then one arrangement is twelve $1 \times 1 \times 1$ cubes, another is one $3 \times 2 \times 2$ solid, another is three $2 \times 2 \times 1$ solids, another is three $4 \times 1 \times 1$ solids, etc. We do not consider, say, $2 \times 2 \times 1$ and $1 \times 2 \times 2$ to be different; these solids are congruent. You may wish to verify, for example, that $B(12) = 11$.

Find, with proof, the integer $m$ such that $10^m < B(2015^{100}) < 10^{m+1}$.

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**–** `http://tinyurl.com/adihaya-bamo-notice` These problems were created by the BAMO Organizing Con

The Bay Area Mathematical Olympiad committee can be emailed at bamo@msri.org (`mailto:bamo@msri.org`). Visit their website at bamo.org (`http://bamo.org`).

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