Round 4

www.artofproblemsolving.com/community/c2384800

by jasperE3

– Day 1

Problem 1 In a circle with a radius of 1 is an inscribed hexagon (convex). Prove that if the multiple of all diagonals that connects vertices of neighboring sides is equal to 27 then all angles of hexagon are equals.

V. Petkov, I. Tonov

Problem 2 Find all polynomials $p(x)$ satisfying the condition:

$$p(x^2-2x)=p(x-2)^2.$$

Problem 3 In the space is given a tetrahedron with length of the edge 2. Prove that distances from some point M to all of the vertices of the tetrahedron are integer numbers if and only if M is a vertex of tetrahedron.

J. Tabov

– Day 2

Problem 4 Let $0<x_1\le x_2\le\ldots\le x_n$. Prove that

$$\frac{x_1}{x_2}+\frac{x_2}{x_3}+\ldots+\frac{x_{n-1}}{x_n}+\frac{x_n}{x_1}\ge\frac{x_2}{x_1}+\frac{x_3}{x_2}+\ldots+\frac{x_n}{x_{n-1}}+\frac{x_1}{x_n}$$

I. Tonov

Problem 5 It is given a tetrahedron $ABCD$ and a plane α intersecting the three edges passing through D. Prove that α divides the surface of the tetrahedron into two parts proportional to the volumes of the bodies formed if and only if α is passing through the center of the inscribed tetrahedron sphere.

Problem 6 It is given a plane with a coordinate system with a beginning at the point O. $A(n)$, when n is a natural number is a count of the points with whole coordinates which distances to O are less than or equal to n.

(a) Find

$$\ell=\lim_{n\to\infty}\frac{A(n)}{n^2}.$$

(b) For which β $(1 < \beta < 2)$ does the following limit exist?

$$\lim_{n\to\infty} \frac{A(n) - \pi n^2}{n^\beta}$$