Round 4

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by jasperE3

Problem 1 A $6n$-digit number is divisible by 7. Prove that if its last digit is moved to the beginning of the number then the new number is also divisible by 7.

Problem 2 Find all n-tuples of reals $x_1, x_2, \dots, x_n$ satisfying the system:

$$\begin{cases} x_1x_2\cdots x_n = 1 \\ x_1 - x_2x_3\cdots x_n = 1 \\ x_1x_2 - x_3x_4\cdots x_n = 1 \\ \vdots \\ x_1x_2\cdots x_{n-1} - x_n = 1 \end{cases}$$

Problem 3 There are given two intersecting lines g_1, g_2 and a point P in their plane such that $\angle(g1, g2) \neq 90^\circ$. Its symmetrical points on any point M in the same plane with respect to the given lines are M_1 and M_2. Prove that:

(a) the locus of the point M for which the points M_1, M_2 and P lie on a common line is a circle k passing through the intersection point of g_1 and g_2.
(b) the point P is an orthocenter of a triangle, inscribed in the circle k whose sides lie at the lines g_1 and g_2.

Problem 4 Let a_1, b_1, c_1 are three lines each two of them are mutually crossed and aren't parallel to some plane. The lines a_2, b_2, c_2 intersect the lines a_1, b_1, c_1 at the points a_2 in A, C_2, B_1; b_2 in C_1, B, A_2; c_2 in B_2, A_1, C respectively in such a way that A is the perpendicular bisector of B_1C_2, B is the perpendicular bisector of C_1A_2 and C is the perpendicular bisector of A_1B_2. Prove that:

(a) A is the perpendicular bisector of B_2C_1, B is the perpendicular bisector of C_2A_1 and C is the perpendicular bisector of A_2B_1;
(b) triangles $A_1B_1C_1$ and $A_2B_2C_2$ are the same.
