Round 4

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Problem 1 It is given the expression $y=\frac{x^2-2x+1}{x^2-2x+2}$, where x is a variable. Prove that:

(a) if x_1 and x_2 are two values of x, the y_1 and y_2 are the respective values of y only if $x_1<x_2$, $y_1<y_2$;
(b) when x is varying y attains all possible values for which $0\le y<1$.

Problem 2 It is given a circle with center O and radius r. AB and MN are two diameters. The lines MB and NB are tangent to the circle at the points M' and N' and intersect at point A. M'' and N'' are the midpoints of the segments AM' and AN'. Prove that:

(a) the points M,N,N',M' are concyclic.
(b) the heights of the triangle $M''N''B$ intersect in the midpoint of the radius OA.

Problem 3 It is given a cube with sidelength a. Find the surface of the intersection of the cube with a plane, perpendicular to one of its diagonals and whose distance from the centre of the cube is equal to h.

Problem 4 There are given a triangle and some internal point P. x,y,z are distances from P to the vertices A,B and C. p,q,r are distances from P to the sides BC,CA,AB respectively. Prove that:

$$xyz\ge(q+r)(r+p)(p+q).$$

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