Silk Road Mathematics Competiton 2021

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by Rickyminer, Hopeoooooo

1 Given a sequence s consisting of digits 0 and 1. For any positive integer k, define v_k the maximum number of ways in any sequence of length k that several consecutive digits can be identified, forming the sequence s. (For example, if $s = 0110$, then $v_7 = v_8 = 2$, because in sequences 0110110 and 01101100 one can find consecutive digits 0110 in two places, and three pairs of 0110 cannot meet in a sequence of length 7 or 8.) It is known that $v_n < v_{n+1} < v_{n+2}$ for some positive integer n. Prove that in the sequence s, all the numbers are the same.

A. Golovanov

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2 For every positive integer m prove the inquility $|\{\sqrt{m}\} - \frac{1}{2}| \geq \frac{1}{8(\sqrt{m}+1)}$
(The integer part $[x]$ of the number x is the largest integer not exceeding x. The fractional part of the number x is a number $\{x\}$ such that $[x] + \{x\} = x$.)

A. Golovanov

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3 In a triangle ABC, M is the midpoint of the AB. A point B_1 is marked on AC such that $CB = CB_1$. Circle ω and ω_1, the circumcircles of triangles ABC and BMB_1, respectively, intersect again at K. Let Q be the midpoint of the arc ACB on ω. Let B_1Q and BC intersect at E. Prove that KC bisects B_1E.

M. Kungozhin

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4 Integers x, y, z, t satisfy $x^2 + y^2 = z^2 + t^2$and$xy = 2zt$ prove that $xyzt = 0$

Proposed by $M.Abduvaliev$
