2020-2021, served also as Greek JBMO TST since the latter didn't take place

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by parmenides51

1 If positive reals x, y are such that $2(x+y)=1+xy$, find the minimum value of expression

$$A=x+\frac{1}{x}+y+\frac{1}{y}$$

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2 Anna and Basilis play a game writing numbers on a board as follows:
The two players play in turns and if in the board is written the positive integer n, the player whose turn is chooses a prime divisor p of n and writes the numbers $n+p$. In the board, is written at the start number 2 and Anna plays first. The game is won by whom who shall be first able to write a number bigger or equal to 31.
Find who player has a winning strategy, that is who may writing the appropriate numbers may win the game no matter how the other player plays.

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3 Determine whether exists positive integer n such that the number $A=8^n+47$ is prime.

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4 Given a triangleABC with $AB<BC<AC$ inscribed in circle (c). The circle $c(A,AB)$ (with center A and radius AB) interects the line BC at point D and the circle (c) at point H. The circle $c(A,AC)$ (with center A and radius AC) interects the line BC at point Z and the circle (c) at point E. Lines ZH and ED intersect at point T. Prove that the circumscribed circles of triangles TDZ and TEH are equal.
