Peru IMO TST 2020

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–	Day 1
1	Find all pairs (m, n) of positive integers numbers with $m > 1$ such that: For any positive integer $b \le m$ that is not coprime with m, its posible choose positive integers $a_1, a_2, \cdots, a_n$ all coprimes with m such that: $$m + a_1 b + a_2 b^2 + \cdots + a_n b^n$$ Is a perfect power. Note: A perfect power is a positive integer represented by a^k, where a and k are positive integers with $k > 1$
2	Let $ABCDE$ be a convex pentagon with $CD = DE$ and $\angle EDC \neq 2 \cdot \angle ADB$. Suppose that a point P is located in the interior of the pentagon such that $AP = AE$ and $BP = BC$. Prove that P lies on the diagonal CE if and only if area (BCD) + area (ADE) = area (ABD) + area (ABP). (Hungary)
3	Given a positive integer n, let M be the set of all points in space with integer coordinates (a, b, c) such that $0 \le a, b, c \le n$. A frog must go to the point $(0, 0, 0)$ to the point (n, n, n) according to the following rules: • The frog can only jump to points of M. • In each jump, the frog can go from point (a, b, c) to one of the following points: $(a + 1, b, c)$, $(a, b + 1, c)$, $(a, b, c + 1)$, or $(a, b, c - 1)$. • The frog cannot pass through the same point more than once. In how many different ways can the frog achieve its goal?
–	Day 2
4	Find all functions $f : \mathbb{N} \to \mathbb{N}$ such that $$f(a)^{bf(b^2)} \le a^{f(b)^3} \quad \text{for all } a, b \in \mathbb{N}.$$

5 You are given a set of n blocks, each weighing at least 1; their total weight is $2n$. Prove that for every real number r with $0 \leq r \leq 2n-2$ you can choose a subset of the blocks whose total weight is at least r but at most $r+2$.

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6 Find all functions $f:\mathbb{Z}_{>0}\to\mathbb{Z}_{>0}$ such that $a+f(b)$ divides $a^2+bf(a)$ for all positive integers a and b with $a+b>2019$.
