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by parmenides51, mathisreal

–	level 2
1	On a board the numbers $1, 2, 3, \ldots, 98, 99$ are written. One has to mark 50 of them, such that the sum of two marked numbers is never equal to 99 or 100. How many ways one can mark these numbers?
2	Let N be a positive integer; a divisor of N is called *common* if it's great than 1 and different of N. A positive integer is called *special* if it has, at least, two common divisors and it is multiple of all possible differences between any two of their common divisors. Find all special integers.
3	Let ABC be a triangle and D is a point inside of the triangle, such that $\angle DBC = 60^\circ$ and $\angle DCB = \angle DAB = 30^\circ$. Let M and N be the midpoints of AC and BC, respectively. Prove that $\angle DMN = 90^\circ$.
4	At each vertex of a 13-sided polygon we write one of the numbers $1, 2, 3, \ldots, 12, 13$, without repeating. Then, on each side of the polygon we write the difference of the numbers of the vertices of its ends (the largest minus the smallest). For example, if two consecutive vertices of the polygon have the numbers 2 and 11, the number 9 is written on the side they determine. a) Is it possible to number the vertices of the polygon so that only the numbers $3, 4$ and 5 are written on the sides? b) Is it possible to number the vertices of the polygon so that only the numbers $3, 4$ and 6 are written on the sides?
5	Prove that there are 100 distinct positive integers $n_1, n_2, \ldots, n_{99}, n_{100}$ such that $\frac{n_1^3+n_2^3+\cdots+n_{100}^3}{100}$ is a perfect cube.
–	level 1
1	In a forest there are 5 trees A, B, C, D, E that are in that order on a straight line. At the midpoint of AB there is a daisy, at the midpoint of BC there is a rose bush, at the midpoint of CD there is a jasmine, and at the midpoint of DE there is a carnation. The distance between A and E is 28 m; the distance between the daisy and the carnation is 20 m. Calculate the distance between the rose bush and the jasmine.
2	In a 2×8 squared board, you want to color each square red or blue in such a way that on each 2×2 sub-board there are at least 3 boxes painted blue. In how many ways can this coloring be done?

Note. A 2×2 board is a square made up of 4 squares that have a common vertex.

3 In a year that has 365 days, what is the maximum number of "Tuesday the 13th" there can be?

Note: The months of April, June, September and November have 30 days each, February has 28 and all others have 31 days.

4 Facundo and Luca have been given a cake that is shaped like the quadrilateral in the figure.
https://cdn.artofproblemsolving.com/attachments/3/2/630286edc1935e1a8dd9e704ed4c813c90038
png
They are going to make two straight cuts on the cake, thus obtaining 4 portions in the shape of a quadrilateral. Then Facundo will be left with two portions that do not share any side, the other two will be for Luca. Show how they can cut the cuts so that both children get the same amount of cake. Justify why cutting in this way achieves the objective.

5 Bob writes 36 consecutive positive integers in a white paper(in ascending order), next he computes the sum of digits of each one of 36 numbers(in the order) and writes the first 16 results in a red paper and the last 10 results in a blue paper. Determine if Bob can choose the 36 integers, such that the sum of the numbers in the red paper is less than or equal to sum of the numbers in the blue paper.