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– level 2

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- 1** We shall call a positive integer *ascending* if its digits read from left to right they are in strictly increasing order. For example, 458 is ascending and 2339 is not. Find the largest ascending number that is a multiple of 56.
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- 2** Alice writes different real numbers in the board, if  $a, b, c$  are three numbers in this board, least one of this numbers  $a + b, b + c, a + c$  also is a number in the board. What's the largest quantity of numbers written in the board???
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- 3** Let  $ABCD$  be a quadrilateral such that  $\angle ABC = \angle ADC = 90$  and  $\angle BCD < 90$ . Let  $P$  be a point inside of the  $ABCD$  such that  $BCDP$  is parallelogram, the line  $AP$  intersects  $BC$  in  $M$ . If  $BM = 2, MC = 5, CD = 3$ . Find the length of  $AM$ .
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- 4** We consider all 7-digit numbers that are obtained by swapping in all ways Possible digits of 1234567. How many of them are divisible by 7?
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- 5** Ababa plays with a word made up of the letters of his name and has set certain rules:  
If you find an  $A$  followed immediately by a  $B$ , you can substitute  $BAA$  for them.  
If you find two consecutive  $B$ 's, you can delete them.  
If you find three consecutive  $A$ 's, you can delete them.  
Ababa begins with the word  $ABABABAABAAB$ .  
With the above rules, how many letters do you have the shortest word you can come up with?  
Why can't you come up with one more word shorter?

– level 1

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- 1** To each three-digit number, Matías added the number obtained by inverting its digits. For example, he added 729 to the number 927. Calculate in how many cases the result of the sum of Matías is a number with all its digits odd.
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- 2** Is it possible to paint 33 squares on a  $9 \times 9$  game board, so that each row and each column of the board has a maximum of 4 painted squares, but if we also paint any other square a row or column appears that has 5 squares painted?
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- 3** Let  $ABCD$  be a rhombus of sides  $AB = BC = CD = DA = 13$ . On the side  $AB$  construct the rhombus  $BAFC$  outside  $ABCD$  and such that the side  $AF$  is parallel to the diagonal  $BD$  of  $ABCD$ . If the area of  $BAFE$  is equal to 65, calculate the area of  $ABCD$ .

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- 4 Let  $n$  be an even integer greater than 2. On the vertices of a regular polygon with  $n$  sides we can place red or blue chips. Two players,  $A$  and  $B$ , play alternating turns of the next mode: each player, on their turn, chooses two vertices that have no tiles and places on one of them a red chip and in the other a blue chip. The goal of  $A$  is to get three vertices consecutive with tiles of the same color.  $B$ 's goal is to prevent this from happening. To the beginning of the game there are no tiles in any of the vertices. Show that regardless of who starts to play, Player  $B$  can always achieve his goal.
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- 5 We will say that two positive integers  $a$  and  $b$  form a *suitable pair* if  $a + b$  divides  $ab$  (its sum divides its multiplication). Find 24 positive integers that can be distribute into 12 suitable pairs, and so that each integer number appears in only one pair and the largest of the 24 numbers is as small as possible.
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