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–   level 2

1   You have a 4-digit whole number that is a perfect square. Another number is built adding 1 to the unit's digit, subtracting 1 from the ten's digit, adding 1 to the hundred's digit and subtracting 1 from the ones digit of one thousand. If the number you get is also a perfect square, find the original number. It's unique?

2   On a board 4×4 the numbers from 1 to 16 are written, one in each box. Andres and Pablo choose four numbers each. Andrés chooses the biggest of each row and Pablo, the biggest of each column. The same number can be chosen by both. Then they are removed from the board all chosen numbers. What is the greatest value that the sum of the numbers can have what are left on the board?

3   The 2018 inhabitants of a city are divided in two groups: the knights(only speak the truth) and the liars(only speak the lie). The inhabitants sat in a circle and everybody spoke "My two neighbours(in the left and in the right) are liars". After this, one inhabitant got off the circle. The 2017 inhabitants sat again in a circle(not necessarily in the same order), and everybody spoke "None of my two neighbours(in the left and in the right) is of the same group of myself"
Can we determine the group of the inhabitant that got off the city?

4   In a parallelogram $ABCD$, let M be the point on the BC side such that $MC = 2BM$ and let N be the point of side CD such that $NC = 2DN$. If the distance from point B to the line AM is 3, calculate the distance from point N to the line AM.

5   Each point on a circle is colored with one of 10 colors. Is it true that for any coloring there are 4 points of the same color that are vertices of a quadrilateral with two parallel sides (an isosceles trapezoid or a rectangle)?

–   level 1

1   Juan makes a list of 2018 numbers. The first is 1. Then each number is obtained by adding to the previous number, one of the numbers $1, 2, 3, 4, 5, 6, 7, 8$ or 9. Knowing that none of the numbers in the list ends in 0, what is the largest value you can have the last number on the list?

2   A thousand integer divisions are made: 2018 is divided by each of the integers from 1 to 1000. Thus, a thousand integer quotients are obtained with their respective remainders. Which of these thousand remainders is the bigger?

3   Let $ABCDEFGHIJ$ be a regular 10-sided polygon that has all its vertices in one circle with center O and radius 5. The diagonals AD and BE intersect at P and the diagonals AH and BI intersect at Q. Calculate the measure of the segment PQ.

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4   Anna must write 7 positive integers, not necessarily distinct, around a circle such that the following conditions are met: • The sum of the seven numbers equals 36. • If two numbers are neighbours, the difference between the largest and the smallest is equal to 2 or 3.
Find the maximum value of the largest of the numbers that Anna can write.

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5   In each square of a 5×5 board one of the numbers $2, 3, 4$ or 5 is written so that the the sum of all the numbers in each row, in each column and on each diagonal is always even. How many ways can we fill the board?

Clarification. A 5×5 board has exactly 18 diagonals of different sizes. In particular, the corners are size 1 diagonals.