Malaysia National Olympiad 2019

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–	Sulong
1	Evaluate the following sum $$\frac{1}{\log_2 \frac{1}{7}}+\frac{1}{\log_3 \frac{1}{7}}+\frac{1}{\log_4 \frac{1}{7}}+\frac{1}{\log_5 \frac{1}{7}}+\frac{1}{\log_6 \frac{1}{7}}-\frac{1}{\log_7 \frac{1}{7}}-\frac{1}{\log_8 \frac{1}{7}}-\frac{1}{\log_9 \frac{1}{7}}-\frac{1}{\log_{10} \frac{1}{7}}$$
3	A factorian is defined to be a number such that it is equal to the sum of it's digits' factorials. What is the smallest three digit factorian?
4	Let $A=\{1,2,...,100\}$ and $f(k), k \in N$ be the size of the largest subset of A such that no two elements differ by k. How many solutions are there to $f(k)=50$?
5	In a triangle ABC, point D lies on AB. It is given that $AD=25, BD=24, BC=28, CD=20. AC=?$
6	It is known that $2018(2019^{39}+2019^{37}+...+2019)+1$ is prime. How many positive factors does $2019^{41}+1$ have?
B1	Given three nonzero real numbers a,b,c, such that $a>b>c$, prove the equation has at least one real root. $$\frac{1}{x+a}+\frac{1}{x+b}+\frac{1}{x+c}-\frac{3}{x}=0$$ @below sorry, I believe I fixed it with the added constraint.
B2	Given a parallelogram $ABCD$, a point M is chosen such that $\angle DAC=\angle MAC$ and $\angle CAB=\angle MAB$. Prove $\frac{AM}{BM}=\left(\frac{AC}{BD}\right)^2$
B3	An arithmetic sequence of five terms is considered $good$ if it contains 19 and 20. For example, $18.5, 19.0, 19.5, 20.0, 20.5$ is a $good$ sequence. For every $good$ sequence, the sum of its terms is totalled. What is the total sum of all $good$ sequences?