



Paraguay Mathematical Olympiad 2006

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- 1 What are the last two digits of the decimal representation of 21^{2006} ?

- 2 Consider all right triangles with integer sides such that the length of the hypotenuse and one of the two sides are consecutive. How many such triangles exist?

- 3 Let $\Gamma_A, \Gamma_B, \Gamma_C$ be circles such that Γ_A is tangent to Γ_B and Γ_B is tangent to Γ_C . All three circles are tangent to lines L and M , with A, B, C being the tangency points of M with $\Gamma_A, \Gamma_B, \Gamma_C$, respectively. Given that $12 = r_A < r_B < r_C = 75$, calculate:
 - a) the length of r_B .
 - b) the distance between point A and the point of intersection of lines L and M .

- 4 Consider all pairs of positive integers (a, b) , with $a < b$, such that
$$\sqrt{a} + \sqrt{b} = \sqrt{2,160}$$
Determine all possible values of a .

- 5 Let ABC be a triangle, and let P be a point on side BC such that $\frac{BP}{PC} = \frac{1}{2}$. If $\angle ABC = 45^\circ$ and $\angle APC = 60^\circ$, determine $\angle ACB$ without trigonometry.