Paraguay Mathematical Olympiad 2013

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1   Evaluate the following expression:

$$2013^2 + 2011^2 + \ldots + 5^2 + 3^2 - 2012^2 - 2010^2 - \ldots - 4^2 - 2^2$$

2   Let ABC be a triangle with area 9, and let M and N be the midpoints of sides AB and AC, respectively.
Let P be the point in side BC such that $PC = \frac{1}{3}BC$. Let O be the intersection point between PN and CM.
Find the area of the quadrilateral $BPOM$.

3   We divide a natural number N, that has k digits, by 19 and get a residue of $10^{k-2} - Q$, where Q is the quotient and $Q < 101$. Also, $10^{k-2} - Q$ is larger than 0. How many possible values of N are there?

4   Pedro and Juan are playing the following game: $-$ There are 2 piles of rocks, with X rocks in one pile and Y rocks in the other pile ($X < 12, Y < 11$). $-$ Each player can draw:
$-$ 1 rock from one of the piles, or
$-$ 2 rocks from one of the piles, or
$-$ 1 rock from each pile, or
$-$ 2 rock from one pile and 1 from the other pile.
Each player must perform one of these four operations in their turns.
The looser is the one who takes the last rock.
Pedro plays first and has a winning strategy.
What are the three maximum possible values of $(X + Y)$?

5   Let ABC be an obtuse triangle, with AB being the largest side.
Draw the angle bisector of $\angle BAC$. Then, draw the perpendiculars to this angle bisector from vertices B and C, and call their feet P and Q, respectively. D is the point in the line BC such that $AD \perp AP$.
Prove that the lines AD, BQ and PC are concurrent.