Austrian Mathematical Olympiad Junior Regional Competition 2021

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by parmenides51

1   The pages of a notebook are numbered consecutively so that the numbers 1 and 2 are on the second sheet, numbers 3 and 4, and so on. A sheet is torn out of this notebook. All of the remaining page numbers are addedand have sum 2021.
(a) How many pages could the notebook originally have been?
(b) What page numbers can be on the torn sheet?

(Walther Janous)

2   A triangle ABC with circumcenter U is given, so that $\angle CBA = 60^o$ and $\angle CBU = 45^o$ apply. The straight lines BU and AC intersect at point D. Prove that $AD = DU$.

(Karl Czakler)

3   The eight points $A, B, ..., G$ and H lie on five circles as shown. Each of these letters are represented by one of the eight numbers $1, 2, ..., 7$ and 8 replaced so that the following conditions are met:
(i) Each of the eight numbers is used exactly once.
(ii) The sum of the numbers on each of the five circles is the same.
How many ways are there to get the letters substituted through the numbers in this way?

(Walther Janous)

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`png`

4   Let p be a prime number and let m and n be positive integers with $p^2 + m^2 = n^2$.
Prove that $m > p$.

(Karl Czakler)