Paraguay Mathematical Olympiad 2020

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1   José has the following list of numbers: $100, 101, 102, ..., 118, 119, 120$. He calculates the sum of each of the pairs of different numbers that you can put together. How many different prime numbers can you get calculating those sums?

2   Laura is putting together the following list: $a_0, a_1, a_2, a_3, a_4, ..., a_n$, where $a_0 = 3$ and $a_1 = 4$. She knows that the following equality holds for any value of n integer greater than or equal to 1:

$$a_n^2 - 2a_{n-1}a_{n+1} = (-2)^n.$$

Laura calculates the value of a_4. What value does it get?

3   In triangle ABC, side AC is 8 cm. Two segments are drawn parallel to AC that have their ends on AB and BC and that divide the triangle into three parts of equal area. What is the length of the parallel segment closest to AC?

4   In the square $ABCD$ the points E and F are marked on the sides AB and BC respectively, in such a way that $EB = 2AE$ and $BF = FC$. Let G be the intersection between DF and EC. If the side of the square equals 10, what is the distance from point G to side AB?

5   The general term of a sequence of numbers is defined as $a_n = \frac{1}{n^2-n}$, for every integer $n \geq 3$. That is, $a_3 = \frac{1}{6}$, $a_4 = \frac{1}{12}$, $a_5 = \frac{1}{20}$, and so on. Find a general expression for the sum S_n, which is the sum of all terms from a_3 until a_n.