Final Round - Costa Rica 2019

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– Day 1

1 In a faraway place in the Universe, a villain has a medal with special powers and wants to hide it so that no one else can use it. For this, the villain hides it in a vertex of a regular polygon with 2019 sides. Olcoman, the savior of the Olcomita people, wants to get the medal to restore peace in the Universe, for which you have to pay 1000 olcolones for each time he makes the following move: on each turn he chooses a vertex of the polygon, which turns green if the medal is on it or in one of the four vertices closest to it, or otherwise red. Find the fewest olcolones Olcoman needs to determine with certainty the position of the medal.

2 Consider the parallelogram $ABCD$, with $\angle ABC = 60$ and sides $AB = \sqrt{3}$, $BC = 1$. Let ω be the circle of center B and radius BA, and let τ be the circle of center D and radius DA. Determine the area of the region between the circumferences ω and τ, within the parallelogram $ABCD$ (the area of the shaded region).
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`png`

3 Let x, y be two positive integers, with $x > y$, such that $2n = x + y$, where n is a number two-digit integer. If $\sqrt{xy}$ is an integer with the digits of n but in reverse order, determine the value of $x - y$

– Day 2

4 Let $g : R \to R$ be a linear function such that $g(1) = 0$. If $f : R \to R$ is a quadratic function such what $g(x^2) = f(x)$ and $f(x + 1) - f(x - 1) = x$ for all $x \in R$. Determine the value of $f(2019)$.

5 We have an a sequence such that $a_n = 2 \cdot 10^{n+1} + 19$. Determine all the primes p, with $p \le 19$, for which there exists some $n \ge 1$ such that p divides a_n.

6 Consider the right isosceles $\triangle\ ABC$ at A. Let L be the intersection of the bisector of $\angle ACB$ with AB and K the intersection point of CL with the bisector of BC. Let X be the point on line AK such that $\angle KCX = 90^o$ and let Y be the point of intersection of CX with the circumcircle of $\triangle\ ABC$. Let Y' the reflection of point Y wrt BC. Prove that $B - K - Y'$.

Notation: $A - B - C$ means than points A, B, C are collinear in that order i.e. B lies between A and C.

– shortlist

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A2 Let $x, y, z \in R$, find all triples (x, y, z) that satisfy the following system of equations:

$2x^2 - 3xy + 2y^2 = 1$ $y^2 - 3yz + 4z^2 = 2$ $z^2 + 3zx - x^2 = 3$

G2 Let H be the orthocenter and O the circumcenter of the acute triangle $\triangle ABC$. The circle with center H and radius HA intersects the lines AC and AB at points P and Q, respectively. Let point O be the orthocenter of triangle $\triangle APQ$, determine the measure of $\angle BAC$.

LR2 A website offers for 1000 colones, the possibility of playing 4 shifts a certain game of randomly, in each turn the user will have the same probability p of winning the game and obtaining 1000 colones (per shift). But to calculate p he asks you to roll 3 dice and add the results, with what p will be the probability of obtaining this sum. Olcoman visits the website, and upon rolling the dice, he realizes that the probability of losing his
money is from $\left(\frac{103}{108}\right)^4$.
a) Determine the probability p that Olcoman wins a game and the possible outcomes with the dice, to get to this one.
b) Which sums (with the dice) give the maximum probability of having a profit of exactly 1000 colones? Calculate this probability and the value of p for this case.

LR3 Consider the following sequence of squares (side 1), in each step the central square is divided into equal parts and colored as shown in the figure:
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png
Square 1 Square 2 Square 3

Let A_n with $n \in N$, $n > 1$ be the shaded area of square n, show that $A_n < \frac{2}{3}$

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