China Second Round Olympiad 2021

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1 Let $k \geq 2$ be an integer and $a_1, a_2, \cdots, a_k$ be k non-zero reals. Prove that there are finitely many pairs of pairwise distinct positive integers $(n_1, n_2, \cdots, n_k)$ such that

$$a_1 \cdot n_1! + a_2 \cdot n_2! + \cdots + a_k \cdot n_k! = 0.$$

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2 In $\triangle ABC$, point M is the middle point of AC. $MD//AB$ and meet the tangent of A to $\odot(ABC)$ at point D. Point E is in AD and point A is the middle point of DE. $\{P\} = \odot(ABE) \cap AC, \{Q\} = \odot(ADP) \cap DM$. Prove that $\angle QCB = \angle BAC$.
`https://z3.ax1x.com/2021/09/12/4pZ7Zj.jpg` (`https://imgtu.com/i/4pZ7Zj`)

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3 If $n \geq 4,\ n \in \mathbb{N}^*,\ n \mid (2^n - 2)$. Prove that $\frac{2^n-2}{n}$ is not a prime number.

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4 Find the minimum value of c such that for any positive integer $n \geq 4$ and any set $A \subseteq \{1, 2, \cdots, n\}$, if $|A| > cn$, there exists a function $f : A \to \{1, -1\}$ satisfying

$$\left| \sum_{a \in A} a \cdot f(a) \right| \leq 1.$$
