AoPS Online
Art of Problem Solving

Final Round - Costa Rica 2011
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–	Level III - Shortlist
–	C2
–	Day 1
1	Let ABC be a triangle with orthocenter H. Let P, Q, R be the reflections of H with respect to sides BC, AC, AB, respectively. Show that H is incenter of PQR.
2	Find the biggest positive integer n such that n is 167 times the amount of it's positive divisors.

3	The archipelago Barrantes - n is a group of islands connected by bridges as follows: there are a main island (Humberto), in the first step I place an island below Humberto and one above from Humberto and I connect these 2 islands to Humberto. I put 2 islands to the left of these 2 new islands and I connect them with a bridge to the island that they have on their right. In the second step I take the last 2 islands and I apply the same process that I applied to Humberto. In the third step I apply the same process to the 4 new islands. We repeat this step n times we reflect the archipelago that we have on a vertical line to the right of Humberto. We connect Humberto with his reflection and so we have the archipelago Barrantes -n. However, the archipelago Barrantes -n exists on a small planet cylindrical, so that the islands to the left of the archipelago are in fact the islands that are connected to the islands on the right. The figure shows the Barrantes archipelago -2, The islands at the edges are still numbered to show how the archipelago connects around the cylindrical world, the island numbered 1 on the left is the same as the island numbered 1 on the right.
	`https://cdn.artofproblemsolving.com/attachments/e/c/803d95ce742c2739729fdb4d74af59d4d0652`
	`png`
	One day two bands of pirates arrive at the archipelago Barrantes - n: The pirates Black Beard and the Straw Hat Pirates. Blackbeard proposes a game to Straw Hat: The first player conquers an island, the next player must conquer an island connected to the island that was conquered in the previous turn (clearly not conquered on a previous shift). The one who cannot conquer any island in his turn loses. Straw Hat decides to give the first turn to Blackbeard. Prove that Straw Hat has a winning strategy for every n.

–	Day 2
4	Let $p_1, p_2, ..., p_n$ be positive real numbers, such that $p_1 + p_2 + ... + p_n = 1$. Let $x \in [0, 1]$ and let

$y_1, y_2, ..., y_n$ be such that $y_1^2 + y_2^2 + ... + y_n^2 = x$. Prove that

$$\left(\sum_{nx \leq k \leq n} y_k \sqrt{p_k}\right)^2 \leq \sum_{k=1}^{n} \frac{k}{n} p_k$$

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5 Given positive integers a, b, c which are pairwise relatively prime, show that

$$2abc - ab - bc - ac$$

is the biggest number that can't be expressed in the form $xbc + yca + zab$ with x, y, z being natural numbers.

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6 Let ABC be a triangle. The incircle of ABC touches BC, AC, AB at D, E, F, respectively. Each pair of the incircles of triangles AEF, BDF, CED has two pair of common external tangents, one of them being one of the sides of ABC. Show that the other three tangents divide triangle DEF into three triangles and three parallelograms.
