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–	level 2
1	Ana and Celia sell various objects and obtain for each object as many euros as objects they sold. The money obtained is made up of some 10 euro bills and less than 10 coins of 1 euro . They decide to distribute the money as follows: Ana takes a 10 euro bill and then Celia, and so on successively until Ana takes the last 10 euro note, and Celia takes all the 1 euro coins . How many euros more than Celia did Ana take? Give all the possibilities. Ana y Celia venden varios objetos y obtienen por cada objeto tantos euros como objetos vendieron. El dinero obtenido está constituido por algunos billetes de 10 euros y menos de 10 monedas de 1 euro. Deciden repartir el dinero del siguiente modo: Ana toma un billete de 10 euros y después Celia, y así sucesivamente hasta que Ana toma el último billete de 10 euros, y Celia se lleva todas las monedas de 1 euro. ¿Cuántos euros más que Celia se llevó Ana? Dar todas las posibilidades.
2	We have a 7x7 board. We want to color some 1x1 squares such that any 3x3 sub-board have more painted 1x1 than no painted 1x1. What is the smallest number of 1x1 that we need to color?
3	Let $ABCDEFGHI$ be a regular polygon of 9 sides. The segments AE and DF intersect at P. Prove that PG and AF are perpendicular.
4	The first 510 positive integers are written on a blackboard: $1, 2, 3, ..., 510$. An *operation* consists of of erasing two numbers whose sum is a prime number. What is the maximum number of operations in a row what can be done? Show how it is accomplished and explain why it can be done in no more operations.
5	If you have 65 points in a plane, we will make the lines that passes by any two points in this plane and we obtain exactly 2015 distinct lines, prove that least 4 points are collinears!!
–	level 1
1	The teacher secretly thought of a three-digit S number. Students A, B, C and D tried to guess, saying, respectively, $541, 837, 291$ and 846. The teacher told them, “Each of you got it right exactly one digit of S and in the correct position ”. What is the number S?
2	6 indistinguishable coins are given, 4 are authentic, all of the same weight, and 2 are false, one is more light than the real ones and the other one, heavier than the real ones. The two false ones

together weigh same as two authentic coins. Find two authentic coins using a balance scale twice only by two plates, no weights.

Clarification: A two-pan scale only reports if the left pan weighs more, equal or less that right.

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3 In the quadrilateral $ABCD$, we have $\angle C$ is triple of $\angle A$, let P be a point in the side AB such that $\angle DPA = 90$ and let Q be a point in the segment DA where $\angle BQA = 90$ the segments DP and CQ intersects in O such that $BO = CO = DO$, find $\angle A$ and $\angle C$.

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4 We say that a number is *superstitious* when it is equal to 13 times the sum of its digits . Find all superstitious numbers.

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5 Twenty-six people gather in a house. Alicia is friends with only one person, Bruno is friends with two people, Carlos is a friend of three, Daniel is four, Elías is five, and so following each person is friend of a person more than the previous person, until reaching Yvonne, the person number twenty-five, who is a friend to everyone. How many people is Zoila a friend of, person number twenty-six?

Clarification: If A is a friend of B then B is a friend of A.
