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–	level 2
1	The road that goes from the town to the mountain cottage is 76 km long. A group of hikers finished it in 10 days, never travelling more than 16 km in two consecutive days, but travelling at least 23 km in three consecutive days. Find the maximum ammount of kilometers that the hikers may have traveled in one day.
2	In a convex quadrilateral $ABCD$, let M, N, P, and Q be the midpoints of AB, BC, CD, and DA respectively. If MP and NQ divide $ABCD$ in four quadrilaterals with the same area, prove that $ABCD$ is a parallelogram.
3	Ana and Luca play the following game. Ana writes a list of n different integer numbers. Luca wins if he can choose four different numbers, a, b, c and d, so that the number $a + b - (c + d)$ is multiple of 20. Determine the minimum value of n for which, whatever Ana's list, Luca can win.
4	In an excavation in ancient Rome an unusual clock with 18 divisions marked with Roman numerals (see figure). Unfortunately the watch was broken into 5 pieces. The sum of the numbers on each piece was the same. Show how he could be broken the clock. https://cdn.artofproblemsolving.com/attachments/7/a/6e83df1bb7adb13305239a152ac95a4a96f55.png
5	Each square on a $n \times n$ board, with $n \geq 3$, is colored with one of 8 colors. For what values of n it can be said that some of these figures included in the board, does it contain two squares of the same color. https://cdn.artofproblemsolving.com/attachments/3/9/6af58460585772f39dd9e8ef1a2d9f375213.png
–	level 1
1	A natural number N is *good* if its digits are $1, 2$, or 3 and all 2-digit numbers are made up of digits located in consecutive positions of N are distinct numbers. Is there a good number of 10 digits? Of 11 digits?
2	Beatriz has three dice on whose faces different letters are written. By rolling all three dice on one table, and choosing each time only the letters of the faces above, she formed the words $$OSA, VIA, OCA, ESA, SOL, GOL, FIA, REY, SUR, MIA, PIO, ATE, FIN, VID.$$

Determine the six letters of each die.

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3 There are nine boxes. In the first there is 1 stone, in the second there are 2 stones, in the third there are 3 stones, and thus continuing, in the eighth there are 8 stones and in the ninth there are 9 stones. The allowed operation is to remove the same number of stones from two different boxes and place them in a third box. The goal is that all stones are in a single box. Describe how to do it with the minimum number of operations allowed.
Explain why it is impossible to achieve it with fewer operations.

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4 Let ABC be a right triangle and isosceles, with $\angle C = 90^o$. Let M be the midpoint of AB and N the midpoint of AC. Let P be such that MNP is an equilateral triangle with P inside the quadrilateral $MBCN$. Calculate the measure of $\angle CAP$

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5 Given 6 balls: 2 white, 2 green, 2 red, it is known that there is a white, a green and a red that weigh 99 g each and that the other balls weigh 101 g each. Determine the weight of each ball using two times a two-plate scale .

Clarification: A two-pan scale only reports if the left pan weighs more than, equal to or less than the right.
