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– level 2

1 A four digit number is called *stutterer* if its first two digits are the same and its last two digits are also the same, e.g. 3311 and 2222 are stutterer numbers. Find all stutterer numbers that are square numbers.

2 The vertices of two regular octagons are numbered from 1 to 8, in some order, which may vary between both octagons (each octagon must have all numbers from 1 to 8). After this, one octagon is placed on top of the other so that every vertex from one octagon touches a vertex from the other. Then, the numbers of the vertices which are in contact are multiplied (i.e., if vertex A has a number x and is on top of vertex A' that has a number y , then x and y are multiplied), and the 8 products are then added.

Prove that, for any order in which the vertices may have been numbered, it is always possible to place one octagon on top of the other so that the final sum is at least 162.

Note: the octagons can be rotated.

3 Let ABC be a triangle such that $\angle ABC = 2\angle BCA$ and $\angle CAB > 90^\circ$. Let M be the midpoint of BC . The line perpendicular to AC that passes through C cuts the line AB at point D . Show that $\angle AMB = \angle DMC$.

4 Six points are given so that there are not three on the same line and that the lengths of the segments determined by these points are all different. We consider all the triangles that they have their vertices at these points. Show that there is a segment that is both the shortest side of one of those triangles and the longest side of another.

5 There are 27 boxes located in a row; each contains at least 12 marbles. The allowed operation is transfer a ball from a box to its neighbor on the right, as long as said neighbor contains more pellets than the box from which the transfer will be made. We will say that a distribution initial of the balls is *happy* if it is possible to achieve, by means of a succession of permitted operations, that all the balls are in the same box. Determine what is the smallest total number of marbles with the that you can have a happy initial layout.

– level 1

1 Pablo says: "I add 2 to my birthday and multiply the result by 2. I add to the number obtained 4 and multiply the result by 5. To the new number obtained I add the number of the month of my birthday (for example, if it's June, I add 6) and I get 342. "

What is Pablo's birthday date? Give all the possibilities

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- 2 We call $S(n)$ the sum of the digits of the integer n . For example, $S(327) = 3 + 2 + 7 = 12$. Find the value of

$$A = S(1) - S(2) + S(3) - S(4) + \dots + S(2011) - S(2012).$$

(A has 2012 terms).

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- 3 From a paper quadrilateral like the one in the figure, you have to cut out a new quadrilateral whose area is equal to half the area of the original quadrilateral. You can only bend one or more times and cut by some of the lines of the folds. Describe the folds and cuts and justify that the area is half.

<https://2.bp.blogspot.com/-btvafZuTvlk/XNY8nba0BmI/AAAAAAAAAKLo/nm4c21A1hAIK3PK1eEwt6F9cds400/may%2B2012%2B11.png>

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- 4 Pedro has 111 blue chips and 88 white chips. There is a machine that for every 14 blue chips, it gives 11 white pieces and for every 7 white chips, it gives 13 blue pieces. Decide if Pedro can achieve, through successive operations with the machine, increase the total number of chips by 33, so that the number of blue chips equals $\frac{5}{3}$ of the amount of white chips. If possible, indicate how to do it. If not, indicate why.

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- 5 There are 12 people such that for every person A and person B there exists a person C that is a friend to both of them. Determine the minimum number of pairs of friends and construct a graph where the edges represent friendships.
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