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## - $\quad$ level 2

1 Find a positive integer $x$ such that the sum of the digits of $x$ is greater than 2011 times the sum of the digits of the number $3 x$ ( 3 times $x$ ).

2 We say that a four-digit number $\overline{a b c d}(a \neq 0)$ is pora if the following terms are true : $\bullet a \geq b \bullet$ $a b-c d=c d-b a$.
For example, 2011 is pora because $20-11=11-02$
Find all the numbers around.
3 In a right triangle rectangle $A B C$ such that $A B=A C, M$ is the midpoint of $B C$. Let $P$ be a point on the perpendicular bisector of $A C$, lying in the semi-plane determined by $B C$ that does not contain $A$. Lines $C P$ and $A M$ intersect at $Q$. Calculate the angles that form the lines $A P$ and $B Q$.

4 Given $n$ points in a circle, Arnaldo write 0 or 1 in all the points. Bernado can do a operation, he can chosse some point and change its number and the numbers of the points on the right and left side of it. Arnaldo wins if Bernado can't change all the numbers in the circle to 0 , and Bernado wins if he can
a) Show that Bernado can win if $n=101$
b) Show that Arnaldo wins if $n=102$

5 Determine for which natural numbers $n$ it is possible to completely cover a board of $n \times n$, divided into $1 \times 1$ squares, with pieces like the one in the figure, without gaps or overlays and without leaving the board. Each of the pieces covers exactly six boxes.
Note: Parts can be rotated.
https://cdn.artofproblemsolving.com/attachments/c/2/d87d234b7f9799da873bebec845c721e4567f png

- level 1

1 The 4 code words
$\square * \otimes \oplus \triangleright$$\bullet$ $\otimes \oslash \oplus$
they are in some order

$$
A M O \text { SUR REO MAS }
$$

Decrypt

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\otimes\oslash\square*\oplus\triangleright\square\bullet\otimes
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2 Using only once each of the digits $1,2,3,4,5,6,7$ and 8 , write the square and the cube of a positive integer. Determine what that number can be.

3 In the rectangle $A B C D, B C=5, E C=1 / 3 C D$ and $F$ is the point where $A E$ and $B D$ are cut. The triangle $D F E$ has area 12 and the triangle $A B F$ has area 27 . Find the area of the quadrilateral $B C E F$.
https://1.bp.blogspot.com/-4w6e729AF9o/XNY9hqHaBaI/AAAAAAAAKLO/eCaNnWmgc7Yj9uV4z29JAvTcW s400/may\%2B2011\%2B11.png

4 Using several white edge cubes of side 1, Guille builds a large cube. Then he chooses 4 faces of the big cube and paints them red. Finally, he takes apart the large cube and observe that the cubes with at least a face painted red is 431 . Find the number of cubes that he used to assemble the large cube.
Analyze all the possibilities.
5 We consider all 14-digit positive integers, divisible by 18 , whose digits are exclusively 1 and 2 , but there are no consecutive digits 2 . How many of these numbers are there?

