Lexington Math Tournament

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by parmenides51, kevinmathz, GammaZero

– Divisions

A1 B9 Ben writes the string

$$\underbrace{111\ldots11}_{\text{2020 digits}}$$

on a blank piece of paper. Next, in between every two consecutive digits, he inserts either a plus sign $(+)$ or a multiplication sign $(\times)$. He then computes the expression using standard order of operations. Find the number of possible distinct values that Ben could have as a result.

Proposed by Taiki Aiba

A2 B6 1001 marbles are drawn at random and without replacement from a jar of 2020 red marbles and n blue marbles. Find the smallest positive integer n such that the probability that there are more blue marbles chosen than red marbles is strictly greater than $\frac{1}{2}$.

Proposed by Taiki Aiba

A3 Find the value of $\lfloor\frac{1}{6}\rfloor + \lfloor\frac{4}{6}\rfloor + \lfloor\frac{9}{6}\rfloor + \cdots + \lfloor\frac{1296}{6}\rfloor$.

Proposed by Zachary Perry

A4 B14 Let $\triangle ABC$ with $AB = AC$ and $BC = 14$ be inscribed in a circle ω. Let D be the point on ray BC such that $CD = 6$. Let the intersection of AD and ω be E. Given that $AE = 7$, find AC^2.

Proposed by Ephram Chun and Euhan Kim

A5 B19 Ada is taking a math test from 12:00 to 1:30, but her brother, Samuel, will be disruptive for two ten-minute periods during the test. If the probability that her brother is not disruptive while she is solving the challenge problem from 12:45 to 1:00 can be expressed as $\frac{m}{n}$, find $m+n$.

Proposed by Ada Tsui

A6 B17 Circle ω has radius 10 with center O. Let P be a point such that $PO = 6$. Let the midpoints of all chords of ω through P bound a region of area R. Find the value of $\lfloor 10R \rfloor$.

Proposed by Andrew Zhao

A7 B15 Let S denote the sum of all rational numbers of the form $\frac{a}{b}$, where a and b are relatively prime positive divisors of 1300. If S can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers, then find $m+n$.

Proposed by Ephram Chun

A8 B12 Find the sum of all positive integers a such that there exists an integer n that satisfies the equation:

$$a! \cdot 2^{\lfloor \sqrt{a} \rfloor} = n!.$$

Proposed by Ivy Zheng

A9 $\triangle ABC$ has a right angle at B, $AB = 12$, and $BC = 16$. Let M be the midpoint of AC. Let ω_1 be the incircle of $\triangle ABM$ and ω_2 be the incircle of $\triangle BCM$. The line externally tangent to ω_1 and ω_2 that is not AC intersects AB and BC at X and Y, respectively. If the area of $\triangle BXY$ can be expressed as $\frac{m}{n}$, compute is $m+n$.

Proposed by Alex Li

A10 B18 Define a sequence $\{a_n\}_{n\geq 1}$ recursively by $a_1 = 1$, $a_2 = 2$, and for all integers $n \geq 2$, $a_{n+1} = (n+1)^{a_n}$. Determine the number of integers k between 2 and 2020, inclusive, such that $k+1$ divides $a_k - 1$.

Proposed by Taiki Aiba

A11 B20 Two sequences of nonzero reals $a_1, a_2, a_3, \dots$ and $b_2, b_3, \dots$ are such that $b_n = \prod_{i=1}^{n} a_i$ and $a_n = \frac{b_n^2}{3b_n - 3}$ for all integers $n > 1$. Given that $a_1 = \frac{1}{2}$, find $|b_{60}|$.

Proposed by Andrew Zhao

A12 Richard comes across an infinite row of magic hats, $H_1, H_2, \dots$ each of which may contain a dollar bill with probabilities $p_1, p_2, \dots$. If Richard draws a dollar bill from H_i, then $p_{i+1} = p_i$, and if not, $p_{i+1} = \frac{1}{2}p_i$. If $p_1 = \frac{1}{2}$ and E is the expected amount of money Richard makes from all the hats, compute $\lfloor 100E \rfloor$.

Proposed by Alex Li

A13 Find the number of integers n from 1 to 2020 inclusive such that there exists a multiple of n that consists of only 5's.

Proposed by Ephram Chun and Taiki Aiba

A14 Two points E and F are randomly chosen in the interior of unit square $ABCD$. Let the line through E parallel to AB hit AD at E_1, the line through E parallel to AD hit CD at E_2, the line through F parallel to AB hit BC at F_1, and the line through F parallel to BC hit AB at F_2. The expected value of the overlap of the areas of rectangles EE_1DE_2 and FF_1BF_2 can be written as $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a+b$.

Proposed by Kevin Zhao

A15 Let x satisfy $x^4+x^3+x^2+x+1=0$. Compute the value of $(5x+x^2)(5x^2+x^4)(5x^3+x^6)(5x^4+x^8)$.

Proposed by Andrew Zhao

A16 Two circles ω_1 and ω_2 have centers O_1 and O_2, respectively, and intersect at points M and N. The radii of ω_1 and ω_2 are 12 and 15, respectively, and $O_1O_2 = 18$. A point X is chosen on segment MN. Line O_1X intersects ω_2 at points A and C, where A is inside ω_1. Similarly, line O_2X intersects ω_1 at points B and D, where B is inside ω_2. The perpendicular bisectors of segments AB and CD intersect at point P. Given that $PO_1 = 30$, find PO_2^2.

Proposed by Andrew Zhao

A17 There are n ordered tuples of positive integers (a,b,c,d) that satisfy

$$a^2+b^2+c^2+d^2 = 13\cdot 2^{13}.$$

Let these ordered tuples be $(a_1,b_1,c_1,d_1),(a_2,b_2,c_2,d_2),\dots,(a_n,b_n,c_n,d_n)$. Compute $\sum_{i=1}^{n}(a_i+b_i+c_i+d_i)$.

Proposed by Kaylee Ji

A18 Let f of degree at most 13 such that $f(k) = 13^k$ for $0\le k\le 13$. Compute the last three digits of $f(14)$.

Proposed by Kaylee Ji

A19 Euhan and Minjune are playing a game. They choose a number N so that they can only say integers up to N. Euhan starts by saying the 1, and each player takes turns saying either $n+1$ or $4n$ (if possible), where n is the last number said. The player who says N wins. What is the smallest number larger than 2019 for which Minjune has a winning strategy?

Proposed by Janabel Xia

A20 Let $ABCD$ be a cyclic quadrilateral with center O with $AB > CD$ and $BC > AD$. Let M and N be the midpoint of sides AD and BC, respectively, and let X and Y be on AB and CD, respectively, such that $AX\cdot CY = BX\cdot DY = 20000$, and $AX\le CY$. Let lines AD and BC hit at P, and let lines AB and CD hit at Q. The circumcircles of $\triangle MNP$ and $\triangle XYQ$ hit at a point R that is on the opposite side of CD as O. Let R_1 be the midpoint of PQ and B, D, and R be collinear. Let O_1 be the circumcenter of $\triangle BPQ$. Let the lines BO_1 and DR_1 intersect at a point I. If $BP\cdot BQ = 823875$, $AB = 429$, and $BC = 495$, then $IR = \frac{a\sqrt{b}}{c}$ where a, b, and c are positive integers, b is not divisible by the square of a prime, and $\gcd(a,c)=1$. Find the value of $a+b+c$.

Proposed by Kevin Zhao

A21 B23 The LHS Math Team wants to play Among Us. There are so many people who want to play that they are going to form several games. Each game has at most 10 people. People are *happy* if they are in a game that has at least 8 people in it. What is the largest possible number of people who would like to play Among Us such that it is impossible to make everyone *happy*?

Proposed by Sammy Charney

A22 B24 In a game of Among Us, there are 10 players and 12 colors. Each player has a "default" color that they will automatically get if nobody else has that color. Otherwise, they get a random color that is not selected. If 10 random players with random default colors join a game one by one, the expected number of players to get their default color can be expressed as $\frac{m}{n}$. Compute $m+n$. Note that the default colors are not necessarily distinct.

Proposed by Jeff Lin

A23 There are 5 people left in a game of Among Us, 4 of whom are crewmates and the last is the impostor. None of the crewmates know who the impostor is. The person with the most votes is ejected, unless there is a tie in which case no one is ejected. Each of the 5 remaining players randomly votes for someone other than themselves. The probability the impostor is ejected can be expressed as $\frac{m}{n}$. Find $m+n$.

Proposed by Sammy Charney

A24 Sam has 1 Among Us task left. He and his task are located at two randomly chosen distinct vertices of a 2021-dimensional unit hypercube. Let E denote the expected distance he has to walk to get to his task, given that he is only allowed to walk along edges of the hypercube. Compute $\lceil 10E \rceil$.

Proposed by Sammy Charney

A25 B27 Alex and Kevin are radish watching. The probability that they will see a radish within the next hour is $\frac{1}{17}$. If the probability that they will see a radish within the next 15 minutes is p, determine $\lfloor 1000p \rfloor$. Assume that the probability of seeing a radish at any given moment is uniform for the entire hour.

Proposed by Ephram Chun

A26 Jeff has planted 7 radishes, labelled R, A, D, I, S, H, and E. Taiki then draws circles through S, H, I, E, D, then through E, A, R, S, and then through H, A, R, D, and notices that lines drawn through SH, AR, and ED are parallel, with $SH = ED$. Additionally, HER is equilateral, and I is the midpoint of AR. Given that $HD = 2$, HE can be written as $\frac{-\sqrt{a}+\sqrt{b}+\sqrt{1+\sqrt{c}}}{2}$, where a, b, and c are integers, find $a+b+c$.

Proposed by Jeff Lin

A27 Ephram is growing 3 different variants of radishes in a row of 13 radishes total, but he forgot where he planted each radish variant and he can't tell what variant a radish is before he picks it. Ephram knows that he planted at least one of each radish variant, and all radishes of one variant will form a consecutive string, with all such possibilities having an equal chance of occurring. He wants to pick three radishes to bring to the farmers market, and wants them to all be of different variants. Given that he uses optimal strategy, the probability that he achieves this can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

Proposed by Jeff Lin

A28 B30 Arthur has a regular 11-gon. He labels the vertices with the letters in $CORONAVIRUS$ in consecutive order. Every non-ordered set of 3 letters that forms an isosceles triangle is a member of a set S, i.e. $\{C, O, R\}$ is in S. How many elements are in S?

Proposed by Sammy Chareny

A29 Find the smallest possible value of n such that $n+2$ people can stand inside or on the border of a regular n-gon with side length 6 feet where each pair of people are at least 6 feet apart.

Proposed by Jeff Lin

A30 A large gathering of people stand in a triangular array with 2020 rows, such that the first row has 1 person, the second row has 2 people, and so on. Every day, the people in each row infect all of the people adjacent to them in their own row. Additionally, the people at the ends of each row infect the people at the ends of the rows in front of and behind them that are at the same side of the row as they are. Given that two people are chosen at random to be infected with COVID at the beginning of day 1, what is the earliest possible day that the last uninfected person will be infected with COVID?

Proposed by Richard Chen

B1 Four Ls are equivalent to three Ms. Nine Ms are equivalent to fourteen T s. Seven T s are equivalent to two W s. If Kevin has thirty-six Ls, how many W s would that be equivalent to?

B2 The area of a square is 144. An equilateral triangle has the same perimeter as the square. The area of a regular hexagon is 6 times the area of the equilateral triangle. What is the perimeter of the hexagon?

B3 Find the number of ways to arrange the letters in $LEXINGTON$ such that the string LEX does not appear.

B4 Find the greatest prime factor of $20!+20!+21!$.

B5 Given the following system of equations

$a_1+a_2+a_3=1$ $a_2+a_3+a_4=2$ $a_3+a_4+a_5=3$ ... $a_{12}+a_{13}+a_{14}=12$ $a_{13}+a_{14}+a_1=13$ $a_{14}+a_1+a_2=14$

find the value of a_{14}.

B7 Zachary tries to simplify the fraction $\frac{2020}{5050}$ by dividing the numerator and denominator by the same integer to get the fraction $\frac{m}{n}$, where m and n are both positive integers. Find the sum of the (not necessarily distinct) prime factors of the sum of all the possible values of $m+n$

B8 In rectangle $ABCD$, $AB=3$ and $BC=4$. If the feet of the perpendiculars from B and D to AC are X and Y , the length of XY can be expressed in the form m/n , where m and n are relatively prime positive integers. Find $m+n$.

B10 In a certain Zoom meeting, there are 4 students. How many ways are there to split them into any number of distinguishable breakout rooms, each with at least 1 student?

B11 $\triangle ABC$ is an isosceles triangle with $AB=AC$. Let M be the midpoint of BC and E be the point on AC such that $AE:CE=5:3$. Let X be the intersection of BE and AM. Given that the area of $\triangle CMX$ is 15, find the area of $\triangle ABC$.

B13 Compute the number of ways there are to completely fill a 3×15 rectangle with non-overlapping 1×3 rectangles

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B15 Let $\triangle AMO$ be an equilateral triangle. Let U and G lie on side AM, and let S and N lie on side AO such that $AU=UG=GM$ and $AS=SN=NO$. Find the value of $\frac{[MONG]}{[USA]}$

B16 Let f be a function $R\to R$ that satisfies the following equation:

$$f(x)^2+f(y)^2=f(x^2+y^2)+f(0)$$

If there are n possibilities for the function, find the sum of all values of $n\cdot f(12)$

B22 A cube has one of its vertices and all edges connected to that vertex deleted. How many ways can the letters from the word "$AMONGUS$" be placed on the remaining vertices of the cube so that one can walk along the edges to spell out "$AMONGUS$"? Note that each vertex will have at most 1 letter, and one vertex is deleted and not included in the walk

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B25 Emmy goes to buy radishes at the market. Radishes are sold in bundles of 3 for $\$5$and bundles of 5 for $\$7$. What is the least number of dollars Emmy needs to buy exactly 100 radishes?

B26 Aidan owns a plot of land that is in the shape of a triangle with side lengths 5,10, and $5\sqrt{3}$ feet. Aidan wants to plant radishes such that there are no two radishes that are less than 1 foot apart. Determine the maximum number of radishes Aidan can plant

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B28 There are 2500 people in Lexington High School, who all start out healthy. After 1 day, 1 person becomes infected with coronavirus. Each subsequent day, there are twice as many newly infected people as on the previous day. How many days will it be until over half the school is infected?

B29 Alicia bought some number of disposable masks, of which she uses one per day. After she uses each of her masks, she throws out half of them (rounding up if necessary) and reuses each of the remaining masks, repeating this process until she runs out of masks. If her masks lasted her 222 days, how many masks did she start out with?

– Guts Round

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1 Find the remainder when $2020!$ is divided by 2020^2.

Proposed by Kevin Zhao

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2 In a five term arithmetic sequence, the first term is 2020 and the last term is 4040. Find the second term of the sequence.

Proposed by Ada Tsui

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3 Circles C_1, C_2, and C_3 have radii $2, 3$, and 6 respectively. If the fourth circle C_4 is the sum of the areas of C_1, C_2, and C_3, compute the radius of C_4.

Proposed by Alex Li

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4 At the Lexington High School, each student is given a unique five-character ID consisting of uppercase letters. Compute the number of possible IDs that contain the string ”LMT”.

Proposed by Alex Li

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5 For what digit d is the base 9 numeral $7d35_9$ divisible by 8?

Proposed by Alex Li

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6 The number 2021 can be written as the sum of 2021 consecutive integers. What is the largest term in the sequence of 2021 consecutive integers?

Proposed by Taiki Aiba

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7 $2020 * N$ is a perfect cube. If N can be expressed as $2^a * 5^b * 101^c$, find the least possible value of $a + b + c$ such that a, b, c are all positive integers and not necessarily distinct.

Proposed by Ephram Chun

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8 A rhombus with sidelength 1 has an inscribed circle with radius $\frac{1}{3}$. If the area of the rhombus can be expressed as $\frac{a}{b}$ for relatively prime, positive a, b, evaluate $a+b$.

Proposed by Alex Li

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9 If $xy : yz : zx = 6 : 8 : 12$, and $x^3 + y^3 + z^3 : xyz$ is $m : n$ where m and n are relatively prime positive integers, then find $m+n$.

Proposed by Ada Tsui

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10 2020 magicians are divided into groups of 2 for the Lexington Magic Tournament. After every 5 days, which is the duration of one match, teams are rearranged so no 2 people are ever on the same team. If the longest tournament is n days long, what is the value of n?

Proposed by Ephram Chun

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11 Cai and Lai are eating cookies. Their cookies are in the shape of 2 regular hexagons glued to each other, and the cookies have area 18 units. They each make a cut along the 2 long diagonals of a cookie; this now makes four pieces for them to eat and enjoy. What is the minimum area among the four pieces?

Proposed by Richard Chen

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12 If the value of the infinite sum

$$\frac{1}{2^2-1^2} + \frac{1}{4^2-2^2} + \frac{1}{8^2-4^2} + \frac{1}{16^2-8^2} + \dots.$$

can be expressed as $\frac{a}{b}$ for relatively prime positive integers a, b, evaluate $a+b$.

Proposed by Alex Li

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13 Let set S contain all positive integers that are one less than a perfect square. Find the sum of all powers of 2 that can be expressed as the product of two (not necessarily distinct) members of S.

Proposed by Alex Li

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14 Ada and Emily are playing a game that ends when either player wins, after some number of rounds. Each round, either nobody wins, Ada wins, or Emily wins. The probability that neither player wins each round is $\frac{1}{5}$ and the probability that Emily wins the game as a whole is $\frac{3}{4}$. If the probability that in a given round Emily wins is $\frac{m}{n}$ such that m and n are relatively prime integers, then find $m+n$.

Proposed by Ada Tsui

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15 $\triangle ABC$ has $AB = 5, BC = 6,$ and $AC = 7$. Let M be the midpoint of BC, and let the circumcircle of $\triangle ABM$ intersect AC at N. If the length of segment MN can be expressed as $\frac{a}{b}$ for relatively prime positive integers a, b find $a + b$.

Proposed by Alex Li

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16 Compute

$$\frac{2019! \cdot 2^{2019}}{(2020^2 - 2018^2)(2020^2 - 2016^2) \dots (2020^2 - 2^2)}.$$

Proposed by Ada Tsui

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17 In a regular square room of side length $2\sqrt{2}$ ft, two cats that can see 2 feet ahead of them are randomly placed into the four corners such that they do not share the same corner. If the probability that they don't see the mouse, also placed randomly into the room can be expressed as $\frac{a-b\pi}{c}$, where a, b, c are positive integers with a greatest common factor of 1, then find $a+b+c$.

Proposed by Ada Tsui

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18 Given that $\sqrt{x+2y} - \sqrt{x-2y} = 2$, compute the minimum value of $x + y$.

Proposed by Alex Li

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19 Find the second smallest prime factor of $18! + 1$.

Proposed by Kaylee Ji

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20 Cyclic quadrilateral $ABCD$ has $AC = AD = 5, CD = 6,$ and $AB = BC$. If the length of AB can be expressed as $\frac{a\sqrt{b}}{c}$ where a, c are relatively prime positive integers and b is square-fre,e evaluate $a + b + c$.

Proposed by Ada Tsui

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21 A sequence with first term a_0 is defined such that $a_{n+1} = 2a_n^2 - 1$ for $n \geq 0$. Let N denote the number of possible values of a_0 such that $a_0 = a_{2020}$. Find the number of factors of N.

Proposed by Alex Li

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22 Find the area of a triangle with side lengths $\sqrt{13}, \sqrt{29},$ and $\sqrt{34}$. The area can be expressed as $\frac{m}{n}$ for m, n relatively prime positive integers, then find $m + n$.

Proposed by Kaylee Ji

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23 Let $f : \mathbb{R}\backslash 0 \to \mathbb{R}\backslash 0$ be a non-constant, continuous function defined such that $f(3^x 2^y) = \frac{y}{x} f(3^y)$ for any $x, y \neq 0$. Compute $\frac{f(1296)}{f(6)}$.

Proposed by Richard Chen and Zachary Perry

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24 In the Oxtingnle math team, there are 5 students, numbered 1 to 5, all of which either always tell the truth or always lie. When Marpeh asks the team about how they did in a 10 question competition, each student i makes 5 separate statements (so either they are all false or all true): "I got problems $i+1$ to $2i$, inclusive, wrong", and then "Student j got both problems i and $2i$ correct" for all $j \neq i$. What is the most problems the team could have gotten correctly?

Proposed by Jeff Lin

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25 Consider the equation $x^4 - 24x^3 + 210x^2 + mx + n = 0$. Given that the roots of this equation are nonnegative reals, find the maximum possible value of a root of this equation across all values of m and n.

Proposed by Andrew Zhao

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26 Let ω_1 and ω_2 be two circles with centers O_1 and O_2. The two circles intersect at A and B. ℓ is the circles' common external tangent that is closer to B, and it meets ω_1 at T_1 and ω_2 at T_2. Let C be the point on line AB not equal to A that is the same distance from ℓ as A is. Given that $O_1O_2 = 15$, $AT_1 = 5$ and $AT_2 = 12$, find $AC^2 + T_1T_2{}^2$.

Proposed by Zachary Perry

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27 A list consists of all positive integers from 1 to 2020, inclusive, with each integer appearing exactly once. Define a move as the process of choosing four numbers from the current list and replacing them with the numbers $1, 2, 3, 4$. If the expected number of moves before the list contains exactly two 4's can be expressed as $\frac{a}{b}$ for relatively prime positive integers, evaluate $a+b$.

Proposed by Richard Chen and Taiki Aiba

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28 13 LHS Students attend the LHS Math Team tryouts. The students are numbered $1, 2, ..13$. Their scores are $s_1, s_2, ...s_{13}$, respectively. There are 5 problems on the tryout, each of which is given a weight, labeled $w_1, w_2, ...w_5$. Each score s_i is equal to the sums of the weights of all problems solved by student i. On the other hand, each weight w_j is assigned to be $\frac{1}{\sum_{s_i}}$, where the sum is over all the scores of students who solved problem j. (If nobody solved a problem, the score doesn't matter). If the largest possible average score of the students can be expressed in the form $\frac{\sqrt{a}}{b}$, where a is square-free, find $a+b$.

Proposed by Jeff Lin

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29 Find the number of pairs of integers (a, b) with $0 \le a, b \le 2019$ where $ax \equiv b \pmod{2020}$ has exactly 2 integer solutions $0 \le x \le 2019$.

Proposed by Richard Chen

30 $\triangle ABC$ has the property that $\angle ACB = 90^\circ$. Let D and E be points on AB such that D is on ray BA, E is on segment AB, and $\angle DCA = \angle ACE$. Let the circumcircle of $\triangle CDE$ hit BC at $F \neq C$, and EF hit AC and DC at P and Q, respectively. If $EP = FQ$, then the ratio $\frac{EF}{PQ}$ can be written as $a + \sqrt{b}$ where a and b are positive integers. Find $a + b$.

Proposed by Kevin Zhao

31 Let real angles $\theta_1, \theta_2, \theta_3, \theta_4$ satisfy

$$\sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \sin\theta_4 = 0,$$
$$\cos\theta_1 + \cos\theta_2 + \cos\theta_3 + \cos\theta_4 = 0.$$

If the maximum possible value of the sum

$$\sum_{i<j} \sqrt{1 - \sin\theta_i \sin\theta_j - \cos\theta_i \cos\theta_j}$$

for $i, j \in \{1, 2, 3, 4\}$ can be expressed as $a + b\sqrt{c}$, where c is square-free and a, b, c are positive integers, find $a + b + c$

Proposed by Alex Li

32 In a lottery there are 14 balls, numbered from 1 to 14. Four of these balls are drawn at random. D'Angelo wins the lottery if he can split the four balls into two disjoint pairs, where the two balls in each pair have difference at least 5. The probability that D'Angelo wins the lottery can be expressed as $\frac{m}{n}$, with m, n relatively prime. Find $m + n$.

Proposed by Richard Chen

33 Let ω_1 and ω_2 be two circles that intersect at two points: A and B. Let C and E be on ω_1, and D and F be on ω_2 such that CD and EF meet at B and the three lines CE, DF, and AB concur at a point P that is closer to B than A. Let Ω denote the circumcircle of $\triangle DEF$. Now, let the line through A perpendicular to AB hit EB at G, GD hit Ω at J, and DA hit Ω again at I. A point Q on IE satisfies that $CQ = JQ$. If $QJ = 36$, $EI = 21$, and $CI = 16$, then the radius of Ω can be written as $\frac{a\sqrt{b}}{c}$ where a, b, and c are positive integers, b is not divisible by the square of a prime, and $\gcd(a, c) = 1$. Find $a + b + c$.

Proposed by Kevin Zhao

34 Your answer to this problem will be an integer between 0 and 100, inclusive. From all the teams who submitted an answer to this problem, let the average answer be A. Estimate the value of $\left\lfloor \frac{2}{3}A \right\rfloor$. If your estimate is E and the answer is A, your score for this problem will be

$$\max\left(0, \lfloor 15 - 2 \cdot |A - E| \rfloor\right).$$

Proposed by Andrew Zhao

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35 Estimate the number of ordered pairs (p, q) of positive integers at most 2020 such that the cubic equation $x^3 - px - q = 0$ has three distinct real roots. If your estimate is E and the answer is A, your score for this problem will be

$$\left\lfloor 15 \min\left(\frac{A}{E}, \frac{E}{A}\right)\right\rfloor.$$

Proposed by Alex Li

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36 Estimate the product of all the nonzero digits in the decimal expansion of $2020!$. If your estimate is E and the answer is A, your score for this problem will be

$$\max\left(0, \left\lfloor 15 - 0.02 \cdot \left|\log_{10}\left(\frac{A}{E}\right)\right|\right\rfloor\right).$$

Proposed by Alex Li
