

**Austria Beginners' Competition 2013**

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by parmenides51

- 1 Find all natural numbers  $n > 1$  for which the following applies:  
The sum of the number  $n$  and its second largest divisor is 2013.

(R. Henner, Vienna)

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- 2 The following figure is given:

<https://cdn.artofproblemsolving.com/attachments/9/b/97a30e248fcd6f098a900c89721a2e1b3b3f0.png>

Determine the number of paths from the starting square  $A$  to the target square  $Z$ , where a path consists of steps from a square to its top or right neighbor square .

(W. Janous, WRG Ursulinen, Innsbruck)

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- 3 Let  $a$  and  $b$  be real numbers with  $0 \leq a, b \leq 1$ . Prove that

$$\frac{a}{b+1} + \frac{b}{a+1} \leq 1$$

When does equality holds?

(K. Czakler, GRG 21, Vienna)

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- 4 Let  $ABC$  be an acute-angled triangle and  $D$  a point on the altitude through  $C$ . Let  $E, F, G$  and  $H$  be the midpoints of the segments  $AD, BD, BC$  and  $AC$ . Show that  $E, F, G$ , and  $H$  form a rectangle.

(G. Anegg, Innsbruck)

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