Austria Beginners' Competition 2010

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by parmenides51

1 Prove that 2010 cannot be represented as the difference between two square numbers.

(B. Schmidt, Graz University of Technology)

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2 In a national park there is a group of sequoia trees, all of which have a positive integer age. Their average age is 41 years. After a 2010 year old building was destroyed by lightning, the average age drops to 40 years. How many trees were originally in the group? At most, how many of them were exactly 2010 years old?

(W. Janous, WRG Ursulinen, Innsbruck)

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3 Let x and y be positive real numbers with $x+y=1$. Prove that

$$\frac{(3x-1)^2}{x}+\frac{(3y-1)^2}{y}\geq 1.$$

For which x and y equality holds?

(K. Czakler, GRG 21, Vienna)

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4 In the right-angled triangle ABC with a right angle at , the side BC is longer than the side AC. The perpendicular bisector of AB intersects the line BC at point D and the line AC at point E. The segments DE has the same length as the side AB. Find the measures of the angles of the triangle ABC?

(R. Henner, Vienna)
