Austria Beginners' Competition 2011

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by parmenides51

1 Let x be the smallest positive integer for which $2x$ is the square of an integer, $3x$ is the third power of an integer, and $5x$ is the fifth power of an integer. Find the prime factorization of x.

(St. Wagner, Stellenbosch University)

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2 Let p and q be real numbers. The quadratic equation

$$x^2 + px + q = 0$$

has the real solutions x_1 and x_2. In addition, the following two conditions apply:
(i) The numbers x_1 and x_2 differ from each other by exactly 1.
(ii) The numbers p and q differ from each other by exactly 1.
Show that then p, q, x_1 and x_2 are integers.

(G. Kirchner, University of Innsbruck)

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3 Let x, y be positive real numbers with $x + y + xy = 3$. Prove that

$$x + y \geq 2.$$

When does equality holds?

(K. Czakler, GRG 21, Vienna)

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4 Let ABC be an isosceles triangle with $AC = BC$. On the arc CA of its circumcircle, which does not contain B, there is a point P. The projection of C on the line AP is denoted by E, the projection of C on the line BP is denoted by F. Prove that the lines AE and BF have equal lengths.

(W. Janous, WRG Ursulincn, Innsbruck)
