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– Day 1

Problem 1 There are $n \geq 2$ coins numbered from 1 to n. These coins are placed around a circle, not necessarily in order.

In each turn, if we are on the coin numbered i, we will jump to the one i places from it, always in a clockwise order, beginning with coin number 1. For an example, see the figure below.

Find all values of n for which there exists an arrangement of the coins in which every coin will be visited.

`https://services.artofproblemsolving.com/download.php?id=YXR0YWNobWVudHMvOC9jL2E0ZDRhNDVh`
`=\&rn=U2NyZWVuIFNob3QgMjAyMS0xMC0wNiBhdCAxNy4xMy4xNS5wbmc=`

Problem 2 Consider the isosceles right triangle ABC with $\angle BAC = 90°$. Let ℓ be the line passing through B and the midpoint of side AC. Let Γ be the circumference with diameter AB. The line ℓ and the circumference Γ meet at point P, different from B. Show that the circumference passing through $A,\ C$ and P is tangent to line BC at C.

Problem 3 Let $\mathbb{R}$ be the set of real numbers. Determine all functions $f : \mathbb{R} \longrightarrow \mathbb{R}$ so that the equality

$$f(x + yf(x + y)) + xf(x) = f(xf(x + y + 1)) + y^2$$

is true for any real numbers x, y.

– Day 2

Problem 4 Lucía multiplies some positive one-digit numbers (not necessarily distinct) and obtains a number n greater than 10. Then, she multiplies all the digits of n and obtains an odd number. Find all possible values of the units digit of n.

Proposed by Pablo Serrano, Ecuador

Problem 5 Celeste has an unlimited amount of each type of n types of candy, numerated type 1, type 2, ... type n. Initially she takes $m > 0$ candy pieces and places them in a row on a table. Then, she chooses one of the following operations (if available) and executes it:

1. She eats a candy of type k, and in its position in the row she places one candy type $k - 1$ followed by one candy type $k + 1$ (we consider type $n + 1$ to be type 1, and type 0 to be type n).

2. She chooses two consecutive candies which are the same type, and eats them.

Find all positive integers n for which Celeste can leave the table empty for any value of m and any configuration of candies on the table.

Proposed by Federico Bach and Santiago Rodriguez, Colombia

Problem 6 Let ABC be a triangle with incenter I, and A-excenter Γ. Let A_1, B_1, C_1 be the points of tangency of Γ with BC, AC and AB, respectively. Suppose IA_1, IB_1 and IC_1 intersect Γ for the second time at points A_2, B_2, C_2, respectively. M is the midpoint of segment AA_1. If the intersection of A_1B_1 and A_2B_2 is X, and the intersection of A_1C_1 and A_2C_2 is Y, prove that $MX = MY$.
