Kurschak Competition 2021

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by MathHorse

1 Let $P_0=(a_0,b_0),P_1=(a_1,b_1),P_2=(a_2,b_2)$ be points on the plane such that $P_0P_1P_2\Delta$ contains the origin O. Show that the areas of triangles P_0OP_1,P_0OP_2,P_1OP_2 form a geometric sequence in that order if and only if there exists a real number x, such that

$$a_0x^2+a_1x+a_2=b_0x^2+b_1x+b_2=0$$

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2 In neverland, there are n cities and n airlines. Each airline serves an odd number of cities in a circular way, that is, if it serves cities $c_1,c_2,\dots,c_{2k+1}$, then they fly planes connecting $c_1c_2,c_2c_3,\dots,c_1c_{2k+1}$. Show that we can select an odd number of cities $d_1,d_2,\dots,d_{2m+1}$ such that we can fly $d_1\to d_2\to\dots\to d_{2m+1}\to d_1$ while using each airline at most once.

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3 Let $A_1B_3A_2B_1A_3B_2$ be a cyclic hexagon such that A_1B_1,A_2B_2,A_3B_3 intersect at one point. Let $C_1=A_1B_1\cap A_2A_3,C_2=A_2B_2\cap A_1A_3,C_3=A_3B_3\cap A_1A_2$. Let D_1 be the point on the circumcircle of the hexagon such that $C_1B_1D_1$ touches A_2A_3. Define D_2,D_3 analogously. Show that A_1D_1,A_2D_2,A_3D_3 meet at one point.
