2021 Indonesia Regional MO

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– Part A

– The test this year was held on 13 September 2021. It consisted of 10 problems, 5 for Part 1 (each problem worth 4 points, or 1 point if the answer is correct but unjustified), and the other 5 for Part 2 (where each problem is worth 7 points). Part 1 takes 60 minutes to complete, whereas part 2 takes 150 minutes to complete. The contest requires complete workings on both parts. Of course you are not allowed to use a calculator, however protractors and set squares are also PROHIBITED.

Part 1 (Speed Round: 60 minutes, 20 points)

Problem 1. Determine the number of ways to distribute 8 distinct storybooks to 3 children, where each child receives at least 2 books.

Problem 2. A point P lies inside of a quadrilateral and the point is connected to the midpoints of all the sides of the quadrilateral, as shown in the figure below. From this construction, the quadrilateral is divided into 4 regions. The areas of three of these regions are written in each of the respective regions. Determine the area of the quadrilateral that is unknown (which is denoted by the question mark).
(The image lies on the attachments in this post!)

Problem 3. Let a, b, c be positive integers, and define $P(x) = ax^2 + bx + c$. Determine the number of triples (a, b, c) such that $a, b, c \leq 10$ and $P(x)$ is divisible by 6 for all positive integers x.

Problem 4. Determine all real solution pairs (x, y) which satisfy the following system of equations:

$$(x^2 + y + 1)(y^2 + x + 1) = 4$$
$$(x^2 + y)^2 + (y^2 + x)^2 = 2.$$

Problem 5. Given a triangle ABC where $\angle ABC = 120^\circ$. Points A_1, B_1, and C_1 lie on segments BC, CA, and AB respectively, so that lines AA_1, BB_1 and CC_1 are the bisectors of the triangle ABC. Determine the measure of $\angle A_1B_1C_1$.

– Part B

– This is the continuation of my previous post, i.e. part 2 of the same Mathematics Olympiad/Competition (Indonesia recently changed its name since 2020's competition). Each problem is worth 7 points and the same rules apply.

Part 2 (Olympiad Round: 150 minutes)

Problem 6. Suta writes 2021 of the first positive integers on a board, such that every number is written exactly once. She then circles some of them, then sums up all the numbers she's circled to get the value K. Then, Suta also adds up all the numbers she didn't circle to obtain that their sum is equal to L. Show that Suta is able to circle some numbers in the beginning, such that $K - L = 2021$.

Problem 7. Determine all natural numbers $n > 3$ such that $\lfloor \sqrt{n} \rfloor - 1$ divides $n + 1$ and $\lfloor \sqrt{n} \rfloor + 1$ divides $n - 1$.

Problem 8. Given a triangle ABC with G as its centroid. Point D is the midpoint of AC. The line passing through G and parallel to BC cuts AB at E. Prove that $\angle AEC = \angle DGC$ if and only if $\angle ACB = 90^\circ$.

Problem 9. (`https://artofproblemsolving.com/community/q2h2671443p23150906`) Let X be the set containing rational positive numbers satisfying both criteria:
(i) If x is rational and $2021 \le x \le 2022$ then $x \in X$.
(ii) If $x, y \in X$, then $\frac{x}{y}$ is also an element of X.
Prove that all positive rational numbers are in X.

Problem 10. Five unit squares from a 9×9 checkerboard are discarded as shown in the figure below (as an attachment for this post). The entire checkerboard will be covered with domino cards so that each domino covers exactly 2 unit squares, and every unit square is covered by exactly 1 domino. Can we tile the checkerboard with dominoes in such a way that every inner vertical and horizontal line (which are not coloured red) cuts at least 2 dominoes?